Production-Based Asset Pricing with Consumption Variety*

Hitesh Doshi
University of Houston

Praveen Kumar
University of Houston

This Version: November 17, 2025

Abstract

We examine the real and financial effects of consumer preferences for variety in a dynamic multisectoral production-based asset pricing model. Our theoretical, quantitative, and empirical analyses show a significant negative relation of industry-level product elasticity of substitution (ES) and equity risk premiums, helping resolve conflicting evidence in the literature. We also utilize real and financial restrictions from our model for GMM estimation of parameters governing recursive CES preferences. We obtain reasonable, efficient estimates of risk-aversion, intertemporal elasticity of substitution (IES), and the ES. Modeling product variety, real structural restrictions, and production-based instruments each contribute to strong identification.

*For helpful comments we thank Franklin Allen, Tim Bresnahan, Tom George, Nils Gottfries, Kris Jacobs, Ravi Jagannathan, Mamoud Medhat, Elisa Pazaj, Valerie Ramey, Enrique Schroth, Jincheng Tong, and Vijay Yerramilli, as well as participants in the American Economic Association Meetings, North American Meetings of the Econometric Society, and the Corporate Policies and Asset Pricing Conference.

1 Introduction

Economic agents generally choose consumption bundles or baskets of various types of consumption goods produced by firms in different industries (or sectors), i.e., there is consumption variety. Indeed, consumption variety is the basic setting for general equilibrium microeconomic theory (Arrow and Debreu, 1954; Arrow and Hahn, 1971). There is a strong intuition that consumer preferences for product variety should affect risk premiums of firms based on their product characteristics, and this may help explain the observed *inter-industry* variation in equity risk-premiums (Fama and French, 1997). Goods with low product differentiation (or high substitutability), such as commodities and basic consumption goods, have greater price elasticity and lower markups compared with highly differentiated goods, such as luxury goods (Berry, Levinsohn and Pakes, 1995; Broda and Weinstein, 2006). In addition, industries with high product differentiation exhibit greater cyclical variation in markups relative to low product differentiation industries (Domowitz, Hubbard and Petersen, 1986).[1] Asset pricing theory then suggests that high product differentiation industries should ceteris paribus exhibit higher equity risk premiums compared with low differentiated goods industries. Furthermore, in production settings, consumer preferences for product variety should affect optimal production and capital investment decisions of value-maximizing firms, and thus influence risk premiums through the real side of the firm.[2] However, there is surprisingly little literature on production-based asset pricing (e.g., Cochrane, 1991; Jermann, 1998) with consumption variety. In this study, we attempt to fill this gap.

The relationship between product substitutability and industry equity risk premium remains open because the sparse, existing literature provides conflicting evidence: Ait Sahelia, Parker and Yogo (2004) find that luxury goods have higher risk premium compared with

[1] This may be due, for example, to greater cyclicality of discretionary expenditures and incomes of high consumption households (Parker and Vissing-Jorgensen, 2009).

[2] This applies to general equilibrium production models with complete markets (Cochrane, 1991) or with incomplete markets (Horvath, 2000). More generally, even with agency problems due to separation of ownership and control (Berle and Means, 1932; Jensen and Meckling, 1976), the preferences of equity owners are not irrelevant for managers.

basic consumption goods industries. In contrast, using procyclical number of industry firms as a proxy for product variety, Bidian et al. (2025) find a positive relation of substitutability and risk premium. Furthermore, the effects of product substitutability on optimal investment and production policies of value-maximizing firms and the attendant implications for asset prices also deserve further study. In particular, can the structural restrictions derived from production-based asset pricing models with consumption variety facilitate joint estimation of parameters that govern preferences for consumption variety, risk aversion and the intertemporal elasticity of substitution (IES)? Our analysis theoretically and empirically helps address these issues.

We analyze a dynamic multisector general equilibrium production-based asset pricing with consumption variety. The representative firm in each sector, or industry uses an industry-specific production technology that stochastically converts capital and materials inputs to output. The representative consumer-investor is endowed with Epstein and Zin's (1989) recursive preferences over streams of constant elasticity of substitution (CES) consumption baskets. We examine the effects of consumer preferences for variety—captured by the intratemporal product elasticity of substitution (ES)—on equilibrium investment, production and equity risk premiums. In addition, we utilize structural restrictions on the representative consumer's consumption-portfolio investment and firms' capital investment and input policies to form moment conditions in generalized method of moments (GMM) estimation of risk aversion, IES, and ES.[3]

Our study makes two main contributions. First, through quantitative analysis utilizing analytic approximations of industry equilibrium paths, as well as cross-sectional analysis of stock returns using a plausible measure of industry ES, we find a significant negative relation of ES and equity risk premiums. To explicate, in our setting, productivity fluctuations are the fundamental shocks to dividends and consumption. Firms' risk premiums, i.e.,

[3] While our multisector model considers consumer goods as well as intermediate goods sectors, for expositional parsimony we focus on consumer goods industries to facilitate comparisons with the received asset pricing and applied economics literatures.

covariance risk, is therefore driven by the sensitivity of their optimal production and investment policies to productivity shocks: Cetris paribus, higher sensitivity reduces fluctuations in dividends and consumption, and lowers risk premiums. Our quantitative analysis shows that the sensitivity of firms' optimal capital investment and output policies to productivity shocks in high ES industries is greater than that in low ES industries (for reasonable parameteric ranges). Intuitively, firms in high ES industries attempt to protect their low markups through a faster adjustment of investment and production to productivity shocks, compared with firms in higher markup, low ES industries. Consequently, equity risk premiums are negatively related to industry ES. Notably, the effect of high ES on risk premiums through this investment and production policy channel works opposite to the effect of procyclical product variety (Bidian et al., 2025). Our quantitative analysis and cross-sectional empirical results show that the former effect appears to dominate the latter in the data. Thus, our analysis helps reconcile and resolve the conflicting results in the literature on the ES-risk premium relation.

Second, we show that structural restrictions derived from industry production equilibrium paths in the presence of consumption variety, and strong IVs based on micro-level production data, substantially enhance identification of salient parameters governing consumer preferences: risk aversion, IES, and the ES. We obtain robust and efficient estimation of these parameters, and the estimates are economically appealing. In comparison, the asset pricing literature often uses equilibrium restrictions from single consumption-good models to estimate risk aversion and IES, while another literature estimates industry ES using imports and consumption data (Feenstra, 1994; Broda and Weinstein, 2006; Redding and Weinstein, 2020). By using moment conditions in our multisector model and utilizing industry production, investment and asset return data, we present a novel estimation framework for the *joint* estimation of risk aversion, IES and industry ES.

For model calibration, our quantitative analysis uses standard sources for aggregate income, consumption, and price data. And for the calibration of industry production processes, we utilize data on consumer goods manufacturing industries from the NBER-CES database of U.S. manufacturing. Our analysis highlights the *interaction* effects of risk aversion, IES and ES on the equilibrium equity risk premium. Intuitively, and as mentioned above, high price-cost markup differentiated goods are exposed to much greater cyclical risk compared to basic consumption goods. More risk averse investors should therefore demand higher compensation for being exposed to greater (negative) covariance of the SDF with the returns of differentiated goods industries, compared with low product differentiation industries; and, similarly, the required risk compensation will depend on the IES. These interaction effects yield several notable findings.

First, for fixed levels of risk aversion and IES, the equilibrium risk premium is negatively related to the product elasticity of substitution (ES). Second, the slope of the negative risk premium-ES relation is increasing in the level of risk aversion, consistent with the intuition above. Third, and in a related vein, the slope of the negative risk premium-ES relation is decreasing in the level of IES. Consequently, the equilibrium risk premium—at any given levels of risk aversion and ES—is highly sensitive to the choice of the ES. For example, risk premium for risk aversion of 10 and IES of 0.2 is 4.67% when ES is around 1.5, but falls to 2.05% when the ES is 3. Hence, the explicit consideration of consumption variety can help explain observed inter-industry variation in equity risk premiums (Fama and French, 1997) for given levels risk aversion and IES of the representative consumer.

In our cross-sectional analysis, we proxy product substitutability by the embedding-based network industry classification developed by Hoberg and Phillips (2024). This measure based on similarity of product characteristics, which is empirically closely related to the notion of product elasticity of substitution.[4] Our sample includes all non-financial Compustat firms

[4]In contrast, Bidian et al. (2025) use number of industry firm as a proxy for product variety, or substitutability. However, the number of industry firms depends on a variety of factors, such as investment opportunities and entry costs, that may be unrelated to product substitutability. For example, the number of firms in the U.S. oil and gas industry jumped by 20% between 2002 and 2012 (U.S. Census data) because of new opportunities from shale oil production. But there was no increase in industry products, namely,

during 1988-2023 with the available concentration measure. Our tests utilize Fama and MacBeth (1973) regressions on firms' stock returns, controlling for the Fama and French (1992, 1993) factors. We find a significant negative relation of product substitutability and expected returns, consistent with our quantitative analysis.

As we mentioned above, we obtain strong identification of risk aversion, IES and ES, i.e., the point estimates are statistically significant and have low dispersion across different specifications, or choice of IVs. The risk aversion estimate clusters around 5, well within the range of risk aversion values considered reasonable in the literature (Mehra and Prescott, 1985). The IES estimates cluster around 0.2 and the tests reject the null of zero IES at probability values below 0.01. The literature reports a wide range of estimates for IES. A long-standing literature finds IES in the low range using aggregate data (Hall, 1988; Campbell, 1999). More recently, using a novel structural estimation approach, based on "mortgage notches" in the UK, Best et al. (2020) also report estimates of IES around 0.1, while a meta-analysis of IES estimates in the literature finds estimates clustered between 0.3-0.4 (Havránek, 2015). However, another strand of the literature reports estimates of IES in excess of 1 (see Bansal and Yaron, 2004). Our estimates of ES for manufactured consumption goods cluster around 2, which is consistent with the estimates in the literature for manufactured goods using import data (Broda and Weinstein, 2006).

To our knowledge, our study is the first to utilize a multisector general equilibrium production-based asset pricing model with recursive preferences to examine the real and financial effects of preferences for consumption variety and to jointly estimate parameters governing consumer preferences with respect to risk aversion, intratemporal product substitution and intertemporal consumption substitution. In doing so, our study contributes to the growing literature of multisectoral equilibrium models that study asset pricing and real outcomes (Papanikolaou, 2011; Kogan and Papanikolaou, 2014; Doshi and Kumar, 2025). Furthermore, by highlighting the significant role of product elasticity of substitution in the

crude oil and natural gas.

determination of equilibrium firm-level capital investment, production and risk premiums, as well as efficient estimation of salient parameters governing consumer preferences, we help integrate the large CES literature in economics (Arrow et al., 1961 and onwards) with the asset pricing literature.

Our study contributes to the consumption-based asset pricing literature. The existing literature highlights attitudes toward risk (parameterized by risk aversion) and intertemporal consumption risks (parameterized by IES). Our analysis indicates that the intratemporal *composition* of the representative consumer's consumption basket based on the demand price elasticities of component goods (parameterized ES) is a significant determinant of equity premiums of industries (or sectors) producing those goods.

The estimation component of our study is related to the large literature that utilizes asset pricing, consumption and investment data to estimate salient consumer preference parameters and asset pricing kernels.[5] Our study adds to this literature by obtaining reliable estimation by using structural restrictions from a production-based asset pricing model incorporating preferences over consumption variety and utilizing information in industry-level production and investment data.

We significantly extend the limited available asset pricing literature on consumption variety. Ait-Sahalia, Parker and Yogo (2004) argue that NIPA consumption weights focus on basic consumption goods. By incorporating luxury good consumption through nonhomothetic preferences, they are able to explain observed equity premium at relatively low levels of risk aversion. Their results are consistent with our more general finding of a negative relation of product elasticity of substitution and equity risk premiums. As we mentioned already, differentiated goods have lower ES compared to basic goods or commodities. Moreover, Ait Sahalia, Parker and Yogo (2004) do not consider the effects of consumption variety on asset pricing in settings with production and capital investment. Meanwhile, Bidian et

[5] The seminal papers in this literature are Hansen and Singleton (1982, 1983). See Cochrane (2005) and Campbell (2018) for helpful discussions of the literature.

al. (2025) consider a model with additive expected utility and use simulated method of moments to estimate the subjective discount factor and risk aversion. With procyclical product variety, they find a positive relation of product substitutability and risk premiums. However, we find that equilibrium value-maximizing capital investment and production can be more sensitive to aggregate shocks in industries with higher product substitutability, facilitating consumption smoothing by investors and ceteris paribus reducing risk premiums. Our quantitative analysis with reasonable calibration and empirical analysis with a comprehensive sample of firms and a measure of substitutability based on product characteristics (rather than number of firms, as in Bidian et al., 2025), finds a negative relation of substitutability and risk premiums. Finally, our estimation of consumer preference parameters includes the IES and is based on U.S. manufacturing data; we also examine the role of real (i.e., investment and production) structural restrictions in the efficient estimation of consumer preference parameters.

We organize the paper as follows. Section 2 introduces the model. Section 3 undertakes the quantitative analysis and Section 4 presents results of the empirical analysis. Sections 5 and 6 undertake structural estimation of parameters, and Section 7 concludes.

2 A multisector general equilibrium model

In a discrete time, infinite horizon setting, we consider an economy consisting of J competitive sectors (or industries), each composed of a continuum of identical firms of unit mass, and producing a different good. It is therefore notationally convenient to exposit the model at the sectoral level. Sectors are partitioned into J_c sectors that produce (final) consumption goods and sectors that produce two types of intermediate goods: J_h sectors that produce material inputs for production and J_k sectors that produce capital inputs.

2.1 Consumption and portfolio investment

There is a continuum of identical consumer-investors (CI) in the economy; the number of CIs is normalized to unity, without loss of generality. The (representative) CI's income each period comprises of dividend payouts from firms, net changes in the value of her/his stock portfolio, and income from a riskless security. Time is discrete. At each t, the CI chooses the consumption vector $\mathbf{c}_t = (c_{1t}, ...c_{Jc,t})$, taking as given the corresponding vector of consumption good prices $\mathbf{p}_t^c$, with the first consumption good serving as the numeraire.

Firms are unlevered, publicly owned and their equity shares trade in frictionless security markets. The CI also has access every period to a (one-period) risk-free security (f) that pays a unit of the numeraire good next period. The mass of risk-free securities is fixed at unity. The CI's asset holdings at the beginning of the period are denoted by the $(J+1)-$dimensional vector $\mathbf{q}_t$. Along with consumption, the CI simultaneously chooses her/his new asset holdings $\mathbf{q}_{t+1}$, taking as given the corresponding (ex-dividend) asset prices $\mathbf{s}_t$. The dividend payouts per share are denoted by $\mathbf{d}_t$.

The representative CI has Epstein and Zin (1989) preferences over intertemporal streams of consumption bundles $\{C_t\}_{t=1}^{\infty}$ that are expressed in recursive form as

$$\mathcal{U}_t = \left[(1-\alpha)C_t^{1-\eta} + \alpha \mathbb{E}_t \left[\mathcal{U}_{t+1}^{1-\gamma}\right]^{\frac{1-\eta}{1-\gamma}}\right]^{\frac{1}{1-\eta}}, \tag{1}$$

when $\gamma \neq 1$ and $\eta \neq 1$. In (1), C_t is the aggregated consumption index with constant elasticity of substitution (CES) among consumption goods, that is,

$$C_t = \left[\sum_{j=1}^{J_c} \phi_j (c_{jt})^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{2}$$

where $\sigma > 1$ is the product elasticity of substitution (ES); $0 < \phi_j < 1$ are the utility weights; α controls the subjective rate of impatience; γ determines the degree of risk aversion; and η^{-1} measures the intertemporal elasticity of substitution (IES) over consumption baskets.

The CI's budget constraint is given by

$$\mathbf{p}_t^c \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t. \tag{3}$$

Because preferences are strictly increasing, the budget constraint (3) will be binding in any optimum and hence W_t also represents the total consumption expenditure at t. In the standard fashion for CES preferences, within-period optimization yields the consumption demand functions (see Internet appendix (A.1))

$$c_{jt}(\mathbf{p}_t^c, W_t) = \frac{W_t}{P_t}\left[\frac{P_t \phi_j}{p_{jt}}\right]^{\sigma} \tag{4}$$

where $P_t = \left[\sum_{j=1}^{J_c} (\phi_j)^{\sigma} (p_{jt})^{1-\sigma}\right]^{1/(1-\sigma)}$ is the aggregate price index. At the optimum, the aggregate real consumption $C_t \equiv C(\mathbf{c}_t) = \frac{W_t}{P_t}$, which is the real income.

Meanwhile, the CI's portfolio optimization equates the real current security price to expected present value of real equity payoffs next period. The real SDF (or pricing kernel) is the intertemporal marginal rate of substitution of real consumption (IMRS). Letting $\Lambda_t \equiv \frac{\partial \mathcal{U}_t}{\partial \mathcal{C}_t} = (1-\alpha) C_t^{-\eta} \mathcal{U}_t^{\eta}$ denote the marginal valuation at t, the SDF for the one-period investment horizon is $\Lambda_{t,t+1} \equiv \frac{\Lambda_{t+1}}{\Lambda_t}$. Using $C_t = \frac{W_t}{P_t}$ and following Epstein and Zin (1989), the real SDF can be written

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \alpha^{\theta} \left(\frac{G_{t+1}^W}{G_{t+1}^P}\right)^{-\eta\theta} R_{C,t+1}^{\theta-1}, \theta \equiv \frac{1-\gamma}{1-\eta}, \tag{5}$$

where G_{t+1}^W and G_{t+1}^P are the gross growth rates in aggregate income and the price index between t and $t+1$, respectively, and $R_{C,t+1}$ is the gross one-period (real) return on an asset that pays aggregate consumption as its dividend.[6] Hence, asset prices satisfy

$$\frac{\mathbf{s}_t}{P_t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\mathbf{d}_{t+1} + \mathbf{s}_{t+1}}{P_{t+1}} \right), \right] \tag{6}$$

which can be expressed in more conventional (or nominal terms) by defining the one-period ahead *nominal* SDF $M_{t,t+1} \equiv P_t \left(\frac{\Lambda_{t,t+1}}{P_{t+1}} \right)$. Since the one-period return between t and $t+1$ in each sector is $R_{j,t+1} = \left(\frac{d_{j,t+1} + s_{j,t+1}}{s_{j,t}} \right)$, and the return on the one-period nominally riskless bond is $R_{f,t+1} = (1/s_{f,t})$, (6) can be written in the standard fashion as the restriction:

$$\mathbf{1} = \mathbb{E}_t \left[M_{t,t+1} \mathbf{R}_{t+1} \right], \tag{7}$$

where $\mathbf{1}$ and $\mathbf{R}_{t+1}$ are the unit and gross nominal returns vectors, respectively.

2.2 Production and dividends

The representative firm in the typical sector produces output Y_t through the production function:[7]

$$F(K_t, H_t, A_t) = A_t (K_t)^{\psi_K} (H_t)^{\psi_H}, \tag{8}$$

where K_t is the firm's capital stock at the beginning of t; H_t are materials input chosen during the period; A_t represents an exogenous, stochastically evolving industry-wide productivity level; and $\psi_K \in (0,1), \psi_H \in (0,1)$ are the output elasticities of capital and inputs, respectively. The sectoral productivity shocks follow a first order log-autoregressive stochastic process, that is,

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}, \tag{9}$$

[6] More precisely, $\frac{\Lambda_{t+1}}{\Lambda_t} = \left(\frac{G^W_{t+1}}{G^P_{t+1}} \right)^{\eta} \left(\frac{U_{t+1}}{\mathbb{E}_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\eta - \gamma}$, which can be shown to equal (5).

[7] For notational ease, we suppress subscripts for sectors and firms unless necessary for exposition.

where ε_{at} is a normal mean zero variable with a stationary variance-covariance matrix (across sectors) Φ_a.[8]

Next, capital stock K_t evolves in the standard fashion, according to

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{10}$$

where δ is the per-period depreciation rate and I_t is the investment at t.

Similar to the literature (Kiyotaki, 1988; Horvath, 2000), we assume that firms in each sector combine intermediate goods to form a composite material input and investment good using the sector-specific CES functions:[9]

$$H_t = \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{jn,t})^{(\zeta_j^h - 1)/\zeta_j^h} \right]^{\zeta_j^h/(\zeta_j^h - 1)}; I_t = \left[\sum_{n=J_h+1}^{J} \varphi_{nj}^k (I_{jn,t})^{(\zeta_j^k - 1)/\zeta_j^k} \right]^{(\zeta_j^k - 1)/\zeta_j^k}, \tag{11}$$

where $H_{jn,t}$ is the quantity of material intermediate good purchased by sector j from sector $n = J_c + 1, ..., J_h$; φ_{nj}^h is the sector-specific weight of this good, and $\zeta_j^h \geq 1$ is the ES among material intermediate goods in sector j. Analogously, one interprets $I_{jn,t}$, φ_{nj}^k and ζ_j^k for investment intermediate goods. The costs of material and investment intermediate goods are $\Upsilon_{jh,t} = \sum_{J_c+1}^{J_h} p_{nt} H_{jn,t}$ and $\Upsilon_{jk,t} = \sum_{J_h+1}^{J} p_{nt} I_{jn,t}$, respectively.

Sectors choose intermediate goods in a two-step process. In the first step, H_t and I_t are determined; in the second stage, conditional on (H_t, I_t), the individual intermediate goods H_{nt} and I_{nt} are chosen to minimize the intermediate cost expenditures Υ_{ht} and Υ_{kt}. This process yields the demand function for intermediate goods as (see Internet appendix (A.2)):

$$H_{nt} = (\varphi_{nj}^h)^{\zeta_j^h} \left[\frac{p_{nt}}{X_t} \right]^{-\zeta_j^h} H_t; I_{nt} = (\varphi_{nj}^k)^{\zeta_j^k} \left[\frac{p_{nt}}{Z_t} \right]^{-\zeta_j^k} I_t, \tag{12}$$

[8] Incorporating industry-specific productivity shocks is consistent with evidence on the significant role of sectoral shocks in aggregate outcomes (Forni and Reichlin, 1998; Foerster, Sarte and Watson, 2011), and is also consistent with our focus on industry risk-premiums. Adding aggregate productivity shocks complicates notation but does not qualitatively affect the analysis of the paper.

[9] The counting notation in (11), the J sectors are partitioned as $\{1, ..., J_c; J_c + 1, ..., J_h; J_h + 1..., J\}$.

where $X_t = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_{nt})^{1-\zeta_j^h}\right]^{1/(1-\zeta_j^h)}$ and $Z_t = \left[\sum_{n=J_h+1}^{J} (\varphi_{nj}^k)^{\zeta_j^k} (p_{nt})^{1-\zeta_j^k}\right]^{1/(1-\zeta_j^k)}$ are the materials and investment intermediate goods price indices for sector. It can be shown that the effective composite material input demand H_t is such that $X_t H_t = \Upsilon_{ht}$, and similarly $Z_t I_t = \Upsilon_{kt}$. Apart from the costs of investment goods, firms are subject to convex capital adjustment costs so that the total investment cost function is

$$O(I_t, K_t) = Z_t I_t + 0.5\upsilon \left(\frac{I_t}{K_t}\right)^2 K_t, \tag{13}$$

where υ is the sector-specific capacity adjustment cost parameter. The process for determination of (H_t, I_t) will be specified below in the characterization of equilibrium.

The number of shares outstanding in each sector at the beginning of t is denoted by Q_t. Net cash flows are paid out as dividends. Then payouts from sector at t are

$$D_t = p_t Y_t - X_t H_t - O(I_t, K_t). \tag{14}$$

Dividends can be negative, which are financed by equity issuance. Without loss of generality, we normalize Q_t to one in each period.

2.3 Equilibrium

The state vector for firms in the typical sector at beginning of t is $\Omega_t = (W_t, P_t, A_t, X_t, Z_t, K_t)$; the first five elements of this vector are taken as exogenous by firms, while K_t is dynamically endogenous. At every t, conditional on Ω_t, the representative firm in each sector is instructed by shareholders to choose $\{H_{t+\tau}, I_{t+\tau}\}_{\tau=0}^{\infty}$ to maximize the conditional present value of real dividends given by[10]

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left(\frac{D_{t+\tau}}{P_{t+\tau}}\right)\right], \tag{15}$$

[10] In general, there will not exist complete contingent markets in this model; hence, the discount rate is given by the representative consumer's marginal utility of real consumption (Brock 1982; Horvath 1998).

subject to the production and capital accumulation constraints specified above.

In equilibrium, firms follow optimal investment and input choice strategies taking as given the prices for the industry good p_t, while consumers follow optimal consumption and portfolio policies represented by (4) and (6). The model is closed by the requirement that product and asset markets clear. It is convenient, from the viewpoint of our empirical analysis, to express firms' objective functions in nominal terms. Defining the nominal SDF for future payoffs at t as $M_{t,t+\tau} \equiv P_t \left(\frac{\Lambda_{t,t+\tau}}{P_{t+\tau}} \right), \tau = 0, 1, ..,$ the objective function in (15) can be re-expressed as $\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau} D_{t+\tau} \right]$. Since $\Lambda_{t,t+\tau} \equiv \frac{\Lambda_{t+\tau}}{\Lambda_t}$ by construction, it will follow that $M_{t,t+\tau} = \frac{M_{t+\tau}}{M_t}, \tau = 0, 1, 2...(M_{t,t} = 1)$. Then, using the Bellman representation, we can define the nominal *cum-dividend* value function of the representative firm along the equilibrium path recursively as a function of the state:

$$V_t(\Omega_t) = \max_{I_t, H_t \geq 0} D_t + \mathbb{E}_t \left[V_{t+1}(\Omega_{t+1}) \right]. \tag{16}$$

The ex-dividend value of the firm will be denoted S_t.

2.4 Equilibrium characterization for consumer goods

We will focus on the asset pricing implication of consumer goods industries (sectors $j = 1, ..., J_c$) for reasons of space and expositional parsimony. There are two principal reasons for this focus. First, the existing empirical asset pricing literature mostly focuses on ERP driven by the (direct) consumption of the representative consumer, so it facilitates intuition on the determinants of sectoral ERP—through comparisons of our results with the literature—to focus on consumer goods industries. Second, the demand functions for consumer goods—and, hence, their sales and dividends—involve the intratemporal ES and sectoral taste parameters (see (4)); these parameters have been estimated by a long literature, which provides a useful benchmark for checking the "reasonableness" of our estimates. In contrast, there is sparse literature on the estimation of the sector-specific CES production and investment parameters ζ^h and ζ^k that drive the demand functions of intermediate goods producers.[11]

We now present the optimality conditions for the representative firm in a consumer goods sector along the equilibrium path. Since the goods markets clear in equilibrium, we can use (4) to solve for the inverted demand functions for consumer goods sectors as

$$\omega_t(Y_t) = \phi \left(W_t P_t^{\sigma-1}\right)^{1/\sigma} (Y_t)^{-1/\sigma} . \tag{17}$$

Intuitively, equilibrium prices for goods are ceteris paribus positively related to the aggregate index, holding fixed the consumption basket. Along an equilibrium path, for each t and state Ω_t, the optimality conditions for (H_t, I_t) are given, respectively, by:[12]

$$p_t F_H(K_t, H_t, A_t) = X_t, \tag{18}$$

$$\begin{aligned} Z_t + \upsilon \left(\frac{I_t}{K_t}\right) \quad = \quad \mathbb{E}_t \Bigg[M_{t,t+1} \Bigg\{ & p_{t+1} F_K(K_{t+1}, H_{t+1}, A_{t+1}) + 0.5\upsilon \left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \\ & (1-\delta)\left(Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\Bigg\} , \end{aligned} \tag{19}$$

where K_{t+1} is given by (10). The goods market clears in each sector so that

$$p_t = \phi \left(\frac{W_t}{P_t}\right)^{1/\sigma} P_t \left[F(K_t, H_t, A_t)\right]^{-1/\sigma} . \tag{20}$$

Finally, the equilibrium dividends D_t then satisfy (14), while the ex-dividend value S_t is

[11] Conceptually, the extension of the equilibrium conditions to intermediate goods sectors in Proposition 1 below is straightforward, since it require substituting the consumer goods demand functions with intermediate goods demand functions from (12). However, extending the quantitative and empirical analysis to study the asset pricing implications of intermediate goods producers is an interesting avenue for future research.

[12] Details of derivations for these conditions are provided in Internet appendix (A.3).

given recursively by

$$\mathbb{E}_t \left[M_{t,t+1} \left(\frac{D_{t+1} + S_{t+1}}{S_t} \right) \right] = 1. \tag{21}$$

Equation (18) reflects the optimality condition that equates marginal cost of inputs (X_t) to the marginal revenue productivity of inputs at the competitive (or market clearing) price. Equation (19) is the Euler condition showing the trade-off between the current marginal cost of investment—represented by the left-hand side—with its discounted expected marginal value—given by or the right hand size.

3 Quantitative analysis

In this section, we utilize our model to quantitatively assess this hypothesis. In addition, we explore the interaction of ES with risk aversion, capital productivity, and adjustment costs in terms of the effects on ERP.

3.1 Analytic approximations

Our study focuses on the effects of preferences for consumption variety (parameterized by σ) on industry-specific equity risk premiums. Hence, we quantitatively examine *industry equilibrium* paths through loglinear approximations around product and asset market equilibrium of our model (as specified in Equations (18)-(21)) in deterministic steady state.[13] In this analysis, the aggregate income and price index (W_t, P_t) as well as the (industry) productivity and input price indices (A_t, X_t, Z_t) are taken as state variables.

The details of the analytic approximations, including the industry and asset market equilibria in the steady state, are given in Internet appendix (B.2.1. and B.2.2). Because the only exogenous shocks to our economy, namely, the sectoral productivity shocks follow a first order log-autoregressive process (see Equation (9)), we will solve the model with

[13] Such partial equilibrium analysis is often utilized in the literature (Veracierto, 2002; Opp, Parlour and Walden, 2014; Doshi and Kumar, 2025).

loglinear approximations of industry and asset markets' equilibrium under the assumption that the aggregate and industry state variables $(W_t, P_t, A_t, X_t, Z_t)$ follow a log-autoregressive system with multivariate normal i.i.d correlated shocks. The model's solution will then give, at any t, the sector capital stock K_{t+1}, product prices p_t, the log of dividends d_t, as well as the log of the SDF m_t and log of equity prices s_t as linear combinations of the industry's state variables.

Specifically, using the notation: $w_t \equiv \log(W_t), \pi_t \equiv \log(P_t)$, $a_t = \log(A_t)$, $x_t = \log(X_t)$, $z_t = \log(Z_t)$, we let $\boldsymbol{\mu}_t \equiv (w_t, \pi_t, a_t, x_t, z_t)'$, $\boldsymbol{\rho} = (\rho_w, \rho_\pi, \rho_a, \rho_x, \rho_z)$, $0 \leq \boldsymbol{\rho} \leq 1$, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{wt,}\varepsilon_{\pi t,}\varepsilon_{at,}, \varepsilon_{xt,}\varepsilon_{zt})'$. Then the log-state vector $\boldsymbol{\mu}_t$ follows the recursive law of motion

$$\boldsymbol{\mu}_{t+1} = \boldsymbol{\rho}' \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_{t+1}. \tag{22}$$

We note that $\boldsymbol{\mu}_t$ includes two aggregate quantities—w_t and π_t—that are common across industries—and three industry-specific quantities: a_t, x_t and z_t. Correspondingly, the shock vector $\boldsymbol{\varepsilon}_t$ is also composed of aggregate and industry-specific shocks. In particular, $\boldsymbol{\varepsilon}_t$ are multi-variate normal mean zero variables with the variance-covariance matrix $\Phi = \left[\Phi_{ij}\right]$, where $Cov(\varepsilon_{wt,}\varepsilon_{wt,}) = \Phi_w^2, Cov(\varepsilon_{\pi t,}\varepsilon_{\pi t,}) = \Phi_\pi^2$ and $Cov(\varepsilon_{wt,}\varepsilon_{\pi t,}) = \Phi_{w\pi}$ are common across , while $Cov(\varepsilon_{wt,}\varepsilon_{at}) = \Phi_{aw}$ etc. are specific to the industry.

3.2 Equity risk premium

Under the maintained assumption of a stationary variance-covariance matrix of the aggregate and industry shocks, it is shown in the Internet appendix (B.2.3) that the unconditional *industry* ERP is a linear multi-factor representation in terms of second moments of aggregate and industry risk factors, namely, $(w_t, \pi_t, a_t, x_t, z_t)$, that are exogenous to the industry. More precisely, the unconditional industry ERP can be written as:

$$\mathbb{E}[r_{t+1} - r_{t+1}^f] = \beta_w \Phi_w^2 + \beta_\pi \Phi_\pi^2 + \beta_{w\pi}\Phi_{w\pi} + \beta_{aw}\Phi_{aw} + \beta_{xw}\Phi_{xw} + \beta_{zw}\Phi_{zw} + \beta_{a\pi}\Phi_{a\pi} + \beta_{x\pi}\Phi_{x\pi} + \beta_{z\pi}\Phi_{z\pi}. \quad (23)$$

In (23), β_w, β_w and $\beta_{w\pi}$ represent the risk loadings of the representative firm in the industry to aggregate shocks, that is, exposure to income and price index shocks. And $\beta_{aw}, \beta_{xw}, \beta_{x\pi}, \beta_{zw}$, and $\beta_{z\pi}$ represent the industry's risk sensitivities to industry-specific shocks due to the covariation between shocks to industry productivity and intermediate cost indices with aggregate shocks. These factor loadings involve the equilibrium (log-linear) dividend and stock price policies, which in turn depend on the optimal capital investment and production policies of the firm.

Two comments are in order regarding the ERP in our model. First, because of their independent effects on product demand, the aggregate consumption basket is not a sufficient statistic for aggregate income and price index with endogenous production. To explicate, note that consumer optimum with CES preferences implies that $C_t = \frac{W_t}{P_t}$, where C_t is the aggregate consumption basket, W_t is the aggregate income and P_t is the (CES) aggregate price index.[14] In particular, with output Y_t, the inverse demand function $\omega_t(Y_t)$ is proportional to $(W_t/P_t)^{1/\sigma} P_t (Y_t)^{-1/\sigma}$, where σ is the ES. Hence, the industry demand curve shifts outward with exogenous increases in aggregate income *or* the price index—that is, C_t does not subsume the effects of the aggregate risk factors on endogenous production—so that W_t and P_t operate as separate aggregate risk factors. Hence, the risk premium is affected by volatilities of aggregate income and price index, as well as their covariation.

Second, the "real side" of our model specifies the contribution to equilibrium ERP due

[14] Consistent with the CES model, we derive the CES price index as $P_t = W_t/C_t$, using NIPA income and consumption data. Hence, by construction, the variance and covariance of log changes in aggregate income and price index utilized in our analysis imply the (annual) volatility of log per capita consumption growth in the data. In sum, our results are not driven by "large" values of implied consumption growth volatility.

to the covariation between industry-specific productivity and materials costs shocks and aggregate shocks. Hence, the risk premium is also affected by the covariation between sectoral productivity shocks and intermediate goods prices with the aggregate income and price index. For example, a positive covariation between sectoral *input* prices and the aggregate price index (P_t) has a positive risk premium because higher production and investment costs—and hence lower cash flows—tend to coincide with low higher real consumption $\frac{W_t}{P_t}$ and higher stochastic discount factor (SDF).

3.3 The effect of product substitutability

The ERP derivation in (23) allows us to study cross-sectional variation in industry-level risk premiums as a function of given industry characteristics. For the purposes of our study, the effect of *intra-industry* (or, simply, industry) product substitutability, measured by the ES, is of special interest. The literature provides extensions of the basic CES model (with economy-wide ES) to allow industry-based heterogeneity in the ES is well known in the literature (Feenstra, 1994; Broda and Weinstein, 2006). We therefore analyze the effect of industry ES on its equilibrium ERP.

In production settings with risk averse investors, capital investment and production decisions of shareholder value-maximizing firms will optimally respond to fundamental shocks to facilitate consumption smoothing of investors by reducing dividend and consumption fluctuations. Because (sectoral) productivity shocks are the fundamental in our model, it follows that the effect of the ES on the investment and production *sensitivity* to productivity shocks will drive the relation of industry product substitutability and equity risk premiums.

We find (see Internet appendix (B.2.4)) that firms' optimal capital investment and output response, or sensitivity to productivity shocks is *positively* related to industry ES, for consumer preferences and production parametric ranges that are consistent with our maintained assumptions. Specifically, for parametric ranges involving $\sigma > 1$ and production input elasticity $\psi_H < 1$. Both these conditions (i.e., $\sigma > 1$ and $\psi_H < 1$) are needed to ensure that consumers' and firms' optimization problems are strictly concave. Therefore, ceteris paribus, there will be greater adjustment to exogenous shocks in high ES industries compared to low ES industries, generating a negative equilibrium relation of industry ES and ERP.[15] We now turn to quantitative and empirical analysis of the relation of industry ERP and ES.

3.4 Data and Calibration

For calibration of our quantitative model, we need industry data on capital, investment, materials input, sales, and productivity. We take these data from the NBER-CES manufacturing database. The latest data available are for 1958-2018 (annual). However, because industry productivity data are generally available only through 2016, our sample period is 1958-2016. The NBER-CES data are in nominal terms. While deflators for materials costs and investment are provided, the appropriate deflators for output and (especially) capital stock are not apparent. For this reason, we work with the nominal SDF in the Euler condition for investment (Equation 19) and, therefore, also asset returns. Of course, the unconditional ERP is expressed in real terms.

Consistent with the theoretical focus, we restrict attention to consumer goods industries subsample of the NBER-CES database by mapping 1997 North American Industry Classification System (NAICS) codes to four-digit 1987 Standard Industry Classification (SIC) codes.[16] Furthermore, the asset pricing literature highlights the role of durability in expected returns (Yogo, 2008; Gomes, Kogan and Yogo, 2009). We therefore focus on the consumer durable (manufactured) goods sector to calibrate our quantitative analysis. Our sectoral variables are taken as the means of the corresponding variables in the data for consumer goods industries. For sectoral data, we take the cross-sectional means of production data

[15] A useful heuristic is that effect of high (low) ES on the ERP is similar to effect of low (high) real frictions, such as capital adjustment costs. And it is well known that there is a positive relation of adjustment costs and ERP (Jermann, 1998).

[16] The list of consumer goods industry NAICS codes is obtained from Statistics Canada (2021).

across the relevant industry codes.

As the proxies for aggregate income (W) and consumption basket (C), we use per-capita national income and consumption expenditures from the Federal Reserve Bank of St. Louis (FRED). We note that the aggregate price index in the model, P, is not necessarily the CPI, since along the equilibrium path consumption basket $C_t = \frac{W_t}{P_t}$. That is, the appropriate price index is the "ideal" (or CES) price index, which is not generally available (Redding and Weinstein 2020). We therefore use the implied equilibrium P from the income and consumption data. We use the deflators for the material costs (H_t) and investment (I_t) as measures of X_t and Z_t, respectively. We compute the covariances between industry factors (A_t, X_t, Z_t) and the aggregate factors (W_t, P_t)—that contribute to the risk premium—from the data using the log-autoregressive specifications in (22).

Table 1 displays the parameterization for the aggregate quantities (which are common across sectors), as well as the sector-specific parameters. The (annual) volatility of per capita national income shocks (Φ_w) in our sample period is 2.47% , while the volatility of the shocks to log aggregate price index (Φ_π) is very low, that is, 0.76%; and the covariance $\Phi_{w\pi}$ is even lower (4.8e-05). Using the relation $\log C_t = \log W_t - \log P_t$, we obtain (annual) volatility of log consumption growth as 2.39%. By construction (since P is forced to equal W/C), this volatility equals (up to rounding) the annual volatility of per-capita consumption in our sample period. The covariances between the industry and aggregate factors tend to be small. Finally, all stochastic processes are highly persistent, that is, have an autocorrelation coefficient exceeding 0.9, with autocorrelation aggregate income and consumer price index being essentially 1.

We now turn to the calibration of parameters relating to consumer preferences, namely, $(\alpha, \gamma, \eta, \sigma, \phi)$. Because our production data are at an annual frequency, we use 3% annual discount rate, that is $\alpha = 0.97$, consistent with multi-sector general equilibrium models in the literature (Horvath, 2000). There is no consensus in the literature on the other parameters, however. The estimated of percentage of income spent on manufactured consumer goods in the highest income decile countries (which includes U.S.) range from around 0.3 (Duarte and Restuccia 2016) to 0.39 (Duarte 2018). We set ϕ as 0.35 for our manufactured consumer goods sector.

For (γ, η, σ), we utilize the structural estimates from our model using GMM (see Section 5) with our aggregate data and production and investment data for manufactured consumer goods industries. The risk aversion estimates (γ), shown in Table 4 below, are robust around 5. We obtain robust estimates of IES (η^{-1}) of around 0.2. We will discuss this estimate further below in Section 5, but as mentioned before, our estimate is consistent with a strand of the literature on IES estimates. We get reliable estimates of ES (σ) of around 2 for manufactured consumer goods industry. Following Feenstra (1994) and Broda and Weinstein (2006), the interpretation of this sector specific ES is the elasticity of substitution amongst different products in the manufactured consumer goods sector, including between durables and non-durables.[17] The estimated σ reliably exceeds 1, as required for convexity of preferences, and within the range estimates of ES reported in the literature.

Turning to the production side of the model, it is well known that because of varying rates of depreciation for different types of capital (equipment, structures, and intellectual property), estimating depreciation rates is challenging. The literature notes that depreciation rates have been trending upwards because of the increased use of computer equipment and software since this lowers the useful life of capital stock (Oliner 1989). Moreover, the depreciation rates on such equipment have been rising. For example, Gomme and Rupert (2007) note that annual depreciation rates of computer equipment have risen from 15% in 1960-1980 to 40% in 1990s, and give estimates for software depreciation rates of about 50%. Epstein and Denny (1980) estimate the depreciation rate of physical capital (in the first part

[17] This approach is based on extending the basic CES basket specification to allow for nonsymmetric or good-specific ES (Broda and Weinstein, 2006). Using this more flexible specification does not change the theoretical characterization of the equilibrium and, hence, the sectoral asset pricing implications. For notational ease, we use σ to denote the sectoral ES in the manufactured consumer goods sector.

of our sample-period) to be about 13%. Because of increasing use of computerized and higher technology equipment in manufacturing during our sample period, we use an annual depreciation rate of 20%.[18] There is also a wide variation in the literature regarding estimates of the capital adjustment cost parameter υ. In particular, utilizing US plant level data, Cooper and Haltiwanger (2006) find υ of around 10% for a strictly convex adjustment cost function, which is the value we utilize in our computations. Finally, we estimate the production function (8) via GMM using our sectoral production data and estimate $\psi_H = 0.79$. Consistent with our competitive industries assumption, we set $\psi_K = 1 - \psi_H = 0.21$.

In the usual fashion, we take the steady state value of per capita income $\bar{W}$ and aggregate (CES) price index $\bar{P}$ as their sample means. By construction of the P_t time-series, the implied steady state consumption-to-income ratio matches the sample mean and the steady-state consumption level is also close to its sample mean. Similarly, the steady state values $\bar{A}$, $\bar{X}$, and $\bar{Z}$ are calibrated so that the model's steady state per capita output, materials inputs, and capital stock match their corresponding sample means (see Internet appendix (C.1)).

3.5 Asset pricing implications

Figure 1 displays surface plots of the effects on ERP of bivariate variations in the ES (σ) and risk aversion (γ). The figure confirms that, for a fixed σ, ERP is positively related to risk aversion. But the ERP is negatively related to the ES. In the CES setting (see (4)), the sectoral ES measures the demand price elasticity of the industry good; hence, the graphical analysis in Figure 1 indicates a negative relation of ERP and price elasticity.

Conceptually and empirically, goods with low product differentiation, such as commodities and basic consumption goods, have greater demand price elasticity compared with highly differentiated goods, such as luxury goods (Berry, Levinsohn and Pakes, 1995; Broda and Weinstein, 2006). As we mentioned already, high price-cost markup differentiated goods are

[18] This value is also the mean depreciation rate estimated by a detailed study of Canadian manufacturing data (Gellatly et al 2007).

exposed to much greater cyclical risk compared to basic consumption goods. Risk averse investors should therefore demand higher compensation for being exposed to the risk of greater (negative) covariance of the SDF with the returns of differentiated goods industries, compared with low differentiation industries. The graphical analysis in Figure 1 is consistent with this intuition.

Figure 1 also shows interaction effects of risk aversion and ES on the ERP, which are in line with the intuition given above. In particular, the negative relation of ERP and ES increases in magnitude with risk aversion. For example, the ERP gradient with respect to σ (from $\sigma = 3$ to 1.5) is 1.3 when $\gamma = 5$, while this gradient is 2.6 when the $\gamma = 10$. Because of the interaction effects of σ and γ, ERP can be sizeable for γ around 10 and σ around 1.5 (around 4.67%); and conversely the ERP can be relatively small for risk aversion of 10 for high demand elasticity goods (around 2.05%).[19]

Because of the prominent role of the IES in asset pricing with Epstein and Zin (1989) preferences, it is also of interest to examine the interaction of the intratemporal ES between products and the IES (η^{-1}). This graphical analysis is presented in Figure 2. ERP is negatively related to IES (for a given σ) and, of course, we see the negative relation of σ and ERP (for a given IES). Moreover, the negative effect of ES and ERP increases in magnitude as IES falls. For example, the ERP gradient with respect to σ (from $\sigma = 3$ to 1.5) is 1.25 when IES is 0.4 ($\eta = 2.5$), while this gradient is 1.4 when the IES is 0.1 ($\eta = 10$). Overall, the effects of IES are opposite of the effects of risk aversion seen in Figure 1. There, ERP is positively related to risk aversion and the negative effect of ES on ERP is increases with risk aversion. This is consistent with the estimation results in Section 3 where we find an inverse relation of risk aversion and IES.

It is also apparent that our general equilibrium model with multiple consumption goods (or product variety), capital investment and production generates sizeable equity risk-premiums

[19] As a benchmark, the mean annualized equity risk-premium for the durable consumption goods manufacturing during our sample period is 6.71%.

with reasonable calibration of parameters governing consumption preferences, as well as production-related parameters consistent with the data and/or the literature. This analysis is thus consistent with the estimates of consumer preferences parameters in Table 3. The sizeable equity risk-premiums generated by our model for reasonable parameterization complement the existing literature on multisector models with investment and production in the presence of capital adjustment costs (Papanikolaou, 2011).

4 Cross-sectional tests

The quantitative analysis above shows that with reasonable calibration, there is a negative relation of industry-level equity risk premiums and ES, i.e., substitutability among product varieties in the industry. We therefore now examine the product substitutability-risk premium relation empirically through cross-sectional analysis of firm-level returns.

We proxy for product similarity using the embedding-based network industry classification developed by Hoberg and Phillips (2024), obtained from the Hoberg–Phillips Data Library (https://hobergphillips.tuck.dartmouth.edu/tnic_basedata.html). This dataset provides similarity scores for each firm's relationship with other firms, where higher scores indicate greater similarity between companies (closer rivals). Since the data contain pairwise similarity scores for all firm combinations, we compute each firm's median similarity score across all other firms to capture its overall product similarity measure. Firms with high product similarity scores exhibit greater product substitutability, compared with firms with low similarity scores, because the former face a larger number of similar competing products compared to the latter firms. The firm-level product similarity score is thus a direct measure of product substitutability (or ES, σ), which is the parameter of interest in our conceptual framework. This product similarity measure is arguably a closer measure of product substitutability compared with some measures utilized in the literature, such as the number of establishments or industry firms (Bidian et al., 2025). As we mentioned before, the number of industry firms depends on a number of factors, such as investment opportunities in the industry and entry costs, which are unrelated to intra-industry product substitutability.[20]

We estimate predictive Fama–MacBeth regressions of monthly stock returns on the product similarity measure, controlling for firm beta, size, and book-to-market ratio. Firm size is calculated as the market value of equity, and book-to-market is defined as the ratio of book value of equity to market value of equity, following the methodology of Fama and French (1992). Because the product similarity measure is available at annual frequency, we hold the product similarity measure constant within each year using its most recently available value.

Table 2 reports the results of the Fama-MacBeth cross-sectional regressions analyzing the relationship between firms' product similarity scores and equity returns, for various specifications of risk factor controls. We find a consistently significant negative relationship: Firms with higher product similarity to their industry peers earn lower subsequent returns, with coefficients on the product similarity score ranging from -4.6 to -5.6 basis points. The negative effect of product similarity becomes stronger and statistically significant as standard firm-level risk factors, such as beta, size, and book-to-market ratio (Fama and French, 1992), are included. The analysis in Table 2 thus empirically supports the hypothesis that ceteris paribus product substitutability, or the product elasticity of substitution, is negatively related to equity risk premiums.

5 Structural estimation

In this section, we undertake joint estimation of three salient parameters governing consumer preferences, namely (γ, η, σ), through GMM. We focus on these parameters for estimation

[20] For example, the development of technology for oil production from shale in the early part of this century significantly increased investment opportunities for smaller firms. This is because shale production has lower set-up costs compared to conventinal production technologies, such as deep-sea production. Consequently, the number of oil and gas firms in the US increased substantially (by about 20%) between 2002 and 2012 (based on Bureau of Labor Statistics data). But it is apparent that there was no increase in the variety of the industry's basic products, namely, crude oil and natural gas.

parsimony, which enhances (estimation) efficiency.[21] Moreover, the joint estimation of risk aversion, IES and ES is novel and of interest to both finance and economics literatures.

5.1 Moment conditions

Our model provides two "real side" moment conditions from the Euler conditions (18)-(19), as well as conditions from the asset market equilibrium (see (7) or (21)). These conditions form the basis of our estimation, and are specified succinctly below.

$$
\begin{aligned}
0 &= \phi\left(W_t\right)^{1/\sigma}\left(P_t\right)^{\frac{\sigma-1}{\sigma}}\left(Y_t\right)^{-\frac{1}{\sigma}}\left(\frac{\psi_H Y_t}{H_t}\right) - X_t, && (24)\\
0 &= -O_I(I_t, K_t) + \mathbb{E}_t\left[M_{t,t+1}\left\{p_{t+1}F_K(K_{t+1}, H_{t+1}, A_{t+1}) - \right.\right. \\
& \quad \left.\left. O_K(I_{t+1}, K_{t+1}) + (1-\delta)O_I(I_{t+1}, K_{t+1})\right\}\right] = 0, && (25)\\
0 &= \mathbb{E}_t\left[M_{t,t+1}\tilde{\mathbf{R}}_{t+1}\right], && (26)
\end{aligned}
$$

where (from Equation (13)) $O_I(I_t, K_t) = Z_t + \upsilon\left(\frac{I_t}{K_t}\right)$, $O_K(I_t, K_t) = -0.5\upsilon\left(\frac{I_t}{K_t}\right)^2$, and $\tilde{\mathbf{R}}_{t+1}$ is the vector of excess returns. In particular, we use the aggregate or market ERP, $\tilde{R}_t^{\Sigma} \equiv R_t^{\Sigma} - R_{ft}$, the consumer goods manufacturing sector (CM, j) ERP, $\tilde{R}_{jt} = R_{jt} - R_{ft}$, and the excess market returns relative to CM sector returns $\tilde{R}_t^{\Sigma,j} = R_t^{\Sigma} - R_{jt}$. In addition, and similar to Epstein and Zin (1991), we use the real aggregate return R_t^{Σ} as a proxy for R_{Ct}, that is, the gross return on the asset that pays aggregate consumption as its dividend.

5.2 Data and calibration

For our estimation analysis, we require data on asset returns, in addition to the NBER-CES production data. Because preference parameters apply generally to consumption, we

[21] In particular, when estimating the discount factor α, we do not obtain consistent or economically appealing estimates, even when estimating (γ, η, α), with exogenous values of ES. With some specifications, we obtain estimates exceeding 1, similar to other estimation attempts using additive utility (Hansen and Singleton (1983)) and recursive preferences (Epstein and Zin (1991)); or we obtain significantly negative values of the IES, which implies that consumption increases with interest rates.

broaden our sample to include both manufactured consumer durables and non-durables. We compute sectoral annualized monthly returns for as value-weighted portfolio returns of component industries. We take the sectoral annual returns as the value-weighted portfolio returns of all firms (in the sector). In particular, we first compute the value-weighted portfolio returns using monthly data and then compute the calendar year returns using the monthly time series. When needed, we apply the consumption price deflator (CPI) to adjust the returns data to real terms. The market and risk free returns are obtained from Kenneth French's website.

For calibration of other parameters, we maintain the values of ϕ, δ, and υ used in Section 3. For the production elasticities, we follow the same procedure as in Section 3, and find $\psi_H = 0.75$ and, hence, set $\psi_K = 1 - \psi_H = 0.25$.[22]

5.3 IVs and time-series characteristics

Because lagged endogenous variables are natural IVs, the sample own and cross-autocorrelations of endogenous variables in the moment conditions of our model are of particular interest. Table 3 shows very high own and cross-autocorrelations in the industry-level industry investment (I_t) and materials input (H_t) in the CM sector. The high serial correlation in capital investment is also noted elsewhere in the literature (Eberly, Rebelo and Vincent, 2012). Thus, there is a potential here that industry investment and material inputs can be utilized as strong IVs in empirical estimation.

5.4 Estimation with industry production data

We now use orthogonality conditions from the equilibrium path in both real (i.e., capital investment and materials) and financial variables to estimate the vector of unknown parameters (γ, η, σ). The real moment restrictions are given by Equations (24) and (25). With

[22] We note that this capital factor share is close to the factor share of 0.3, which is often assumed in the real business cycle literature.

these two real restrictions, we potentially have 5 moment restrictions when we also include the three asset markets moment conditions used in the previous section, i.e., involving $\tilde{R}_t^{\Sigma}$, $\tilde{R}_{jt}$, and $\tilde{R}_t^{\Sigma,j}$ (where we now make the sector notation explicit).

However, there are well known pitfalls in adding moment conditions with fixed sample size, especially if they include weak moment conditions since increased estimation efficiency comes at the cost of increasing estimator bias (Han and Phillips, 2006; Newey and Windmeijer, 2009). Furthermore, a large number of moment conditions raises the likelihood of mis-specification bias through utilization of possible invalid restrictions (Andrews, 1999). Consequently, our main test design for estimation uses only 4 moment restrictions: the two real moment conditions and two out of the three excess return restrictions involving market ERP or industry ERP or the excess aggregate return over the industry return. However, for robustness we also undertake estimation with all five moment conditions: two from the product market side and three types of excess return restrictions from the asset market side.

Because of the increased number of moment restrictions, we construct the IVs parsimoniously. Hence, we use only lagged returns for the asset market conditions, and lagged control variables—$H_{t-\ell}$ and $I_{t-\ell}$—for the corresponding optimality conditions ((24) and (25), respectively). Collectively, we denote these IVs as $\Pi_N, N = 1, 2, 3, 4$, and we will specify these when discussing the results.

The results of estimating (γ, η, σ) using real optimality conditions and sectoral production data are displayed in Table 4. The first three rows are based on 4 moment conditions that include the aggregate and industry ERP restrictions, along with product market restrictions (24) and (25). The IV $\Pi_1 = \tilde{R}_{t-\ell}^{\Sigma}, \tilde{R}_{j,t-\ell}, \ell = 1, 2, 3; H_{t-\ell}, I_{t-\ell}, \ell = 1, 2, 3$. The IV Π_2 differs from Π_1 by setting $H_{t-\ell}, \ell = 1$, to examine the implications of asymmetry between short run utilization of material inputs versus the long run effects of investment on capital stock. The IV Π_3 differs from Π_2 by using only two year lags for the market risk premium, that is, $\tilde{R}_{t-\ell}^{\Sigma}$, $\ell = 1, 2$. The specification in the fourth row, associated with Π_4, substitutes the moment restriction $\mathbb{E}_t\left[\Gamma^j_{-1}\left\{M_{t,t+1}(R^{\Sigma}_t - R_{jt})\right\}\right] = 0$ for the restriction $\mathbb{E}_t\left[\Gamma^{\Sigma}_{-2}\left\{M_{t,t+1}\tilde{R}^{\Sigma}_t\right\}\right] = 0$ and uses $R^{\Sigma}_{t-1}, R_{j,t-1}$, $I_{t-\ell}, \ell = 1, 2, 3$, and $H_{t-\ell}, \ell = 1$ as IVs. Finally, the specification Π_5 in the fifth row uses all 5 moment restrictions with the (equity return) IVs $\tilde{R}^{\Sigma}_{t-1}, \tilde{R}_{j,t-1}, R^{\Sigma}_{t-1}$ and $R_{j,t-1}$, along with the real IVs utilized in Π_2.

The point estimates of the unknown parameter vector (γ, η, σ) in each specification are statistically significant, with plausible values, and quite robust to changes in IVs and moment restrictions. The estimates of risk aversion (rounded to the nearest whole number) in all specifications are centered around 5. More specifically, four out of the five specifications in Table 4 generate risk aversion estimate of 5, while the other specification yields a risk aversion estimate around 6. Overall, the risk aversion estimates are well within the range of risk aversion values (2-10) considered plausible (or reasonable) by the literature (Mehra and Prescott, 1985).

Next, the IES estimates in Table 4 cluster around 0.2, and the empirical tests reject the null of zero IES at p-values significantly below 0.01. As we mentioned earlier, there is no consensus in the literature on the value of IES. With additive utility and using aggregate data, Hall (1988) finds low estimates of IES that are not significantly different from zero, and sets an upper bound of 0.1. Similarly, Epstein and Zin (1991) and van Binsbergen et al. (2008) estimate parameters of recursive preferences using different approaches and datasets but nevertheless report low estimates of the IES. More recently, using a novel source of quasi-experimental variation in interest rates (to address the challenge of finding exogenous variations in interest rates) and exploiting "notched" mortgage loan schedules in the UK that imply discrete jumps in mortgage rates at critical loan thresholds, Best et al. (2020) use individual home refinancing data to estimate IES around 0.1. More generally, a large number of studies using micro data report IES estimates below 0.4 (Havránek, 2015; Havránek et al., 2015).[23]

[23] However, the literature also reports estimates exceeding 1 (Vissing-Jorgensen (2002), Gruber (2013)).

Note also that the estimates of $\hat{\theta}$ are reliably positive and strictly less than 1. This implies a negative relation of aggregate returns and the SDF, that is, declines in market returns generate increases in the SDF and hence the risk premium, which is empirically appealing. Indeed, this property is present in empirical parameterizations of Epstein and Zin (1989) preferences commonly used in the literature (Bansal and Yaron, 2004; Croce, 2014), where the IES exceeds 1 (that is, $\frac{1}{\eta} > 1$) so that θ is negative (since risk aversion is generally taken to be higher than 1). However, the estimates in Table 4 imply a preference for late resolution of uncertainty because $\hat{\gamma}(1/\hat{\eta}) < 1$, which is consistent with other production and investment based asset pricing studies in the literature (Papanikolau, 2011).[24]

While we undertake our estimation with recursive preferences, where (relative) risk aversion and IES are treated as separate parameters, the majority of the specifications in Table 4 imply an almost exact inverse relationship between these parameters.[25] As is well known, in the case of additive power expected utility, there is an exact inverse relationship between risk aversion and the IES. There exist other instances in the literature where estimates of risk aversion and IES are close to being in an inverse relationship (van Binsbergen et al., 2008), although in different regions of the parameter space.[26] In a related vein, the estimation results imply that consumer risk preferences are not statistically different from a expected power utility specification, similar to Epstein and Zin (1991)—albeit with a different relative risk aversion coefficient.

Turning to the parameters relating to the CES product variety preferences, the estimates of the product elasticity of substitution (ES, σ) for the manufactured consumer goods sector cluster around 2. These estimates significantly exceed 1, which is the requirement of the

[24] Evidence supporting the assumption of preference for early resolution of uncertainty is mixed. While some studies show that parameterization consistent with preference for early resolution can help match aggregate asset pricing moments (e.g., Bansal and Yaron (2004)), cross-sectional studies that examine the relation of risk-premia to investment maturities present confounding evidence: Binsbergen et al. (2012) find that claims to long-maturity dividends carry lower risk premia; and Giglio et al. (2015) and Weber (2018) find negative relation of risk premia and duration of risky cash flows.

[25] In the case of specification Π_3, the risk aversion is strictly greater than the inverse of the IES.

[26] van Binsbergen et al. (2008) estimate the IES as 0.06 and risk aversion as 46 (whose inverse is 0.02).

theoretical model. The estimation of the CES in Dixit and Stiglitz (1977) preferences at the industry level is of long-standing interest in the applied economics literature. Broda and Weinstein (2006) use import data on product varieties from 1972-1988 and 1990-2001 and report median ES estimates for differentiated goods (at the four-digit Standard International Trade Classification (SITC) level) of 2.1 for 1990-2001 (and 2.5 for 19972-1988). Hottman and Monarch (2018) use import data from 1998 to 2014 and report the median ES for tradable consumer goods (at the four digit NAICS level) of 2.75. Thus, our estimates appear reasonably consistent with estimates of sectoral ES in the literature.

In sum, the results in Table 4 show that a system of moment conditions and IVs derived from a multisectoral general equilibrium production-basesd asset pricing model with explicit modeling of product variety lead to strong identification of salient parameters governing consumer preferences. In particular, the multisectoral set up allows us to utilize industry-level real (i.e., production and capital investment) moment restrictions as well as IVs based on (industry-level) production and investment data. The resultant estimates of the parameters are statistically significant, have low dispersion across IVs, and are appealing from a variety of perspectives. Moreover, the estimates in Table 4 imply that the model (in Section 2) is consistent with the industry-level equity returns in the data with reasonable values of risk aversion, IES, and ES.

6 Euler conditions, IVs and estimation efficiency

There is strong identification of consumer preference parameters with economically reasonable values in Table 4. Relative to received literature on estimation of consumer preference parameters, the estimation in Table 4 has three notable aspects. First, we explicitly incorporate consumer preferences over product variety through the ES, σ, in the estimation. Second, we add industry-level real moment conditions (24) and (25). Third, we use industry production data in the IVs, which helps exploit their stronger own- and cross-autocorrelations as seen in Table 1. It is useful to examine the effects of these three changes in enhancing the identification evident in Table 4.

6.1 Effects of product variety and industry Euler conditions

We first examine whether the improved estimation of risk aversion and IES in Table 4 is an artefact of raising the number of estimated parameters from 2, i.e., (γ, η), to 3, i.e., (γ, η, σ). The alternative hypothesis is that incorporating preferences over product variety (ES) in the canonical asset pricing model adds structural restrictions—i.e., derived from the equilibrium conditions of our model—that enhance identification of all the parameters of interest, in particular, (γ, η). To disentangle these two hypotheses, we undertake estimation by using the moment conditions and IVs in Π_1, but fixing the value of $\sigma = 2.03$, consistent with the estimate in Table 4. As seen in the first row of Table 5, we find reliable estimates of (γ, η) essentially identical to that in Table 4 (for Π_1). We conclude that it is the incorporation of preferences over product variety, rather than the higher number of estimated parameters per se, that improves identification of risk aversion and IES.

We now turn to the role of the moment conditions derived from the industry equilibrium. Improved identification or estimation efficiency in Table 4 could arise from the use of one or both of the Euler conditions. We therefore examine the relative contributions of the Euler condition for investment and the materials input Euler condition. To facilitate comparison with Table 4, we maintain the IVs for moment conditions on returns given in Π_1. (We label the specifications with "bars" to reflect constrained estimation.) We first use only the investment Euler condition, along with the asset returns moment conditions and IVs in Π_1. We are unable to reliably estimate all three parameters (γ, η, σ). However, using $\sigma = 2.03$ and estimating only (γ, η) generates estimates similar to Table 4, as seen in the second row of Table 5. We conclude that adding the intertemporal investment Euler equation significantly enhances identification of risk aversion and IES *conditional* on the ES.

Next, the third row of Table 5 presents estimation results when we include the materials optimality condition but exclude the investment Euler condition. In this case, we obtain a reliable estimate of ES, which is consistent with the estimation in Table 4. However, the risk aversion and IES estimates are highly distorted relative to the estimates in Table 4—the former being too high and the latter being too low, and there is also a significant decline in estimation precision of θ. We conclude that estimation efficiency and/or the economic appeal of estimates appear to significantly deteriorate if either of the two real conditions (i..e, the investment Euler and the materials optimality conditions) are excluded.

The fourth row of Table 5 shows estimates from utilizing the materials input and investment optimality conditions, while excluding the asset returns moment condition. The estimates are significant and close to the estimates in Table 4. However, the risk aversion estimate is lower than the robust estimate in Table 4. These results suggest that capital investment and production data serve as strong IVs and significantly enhance identification of parameters governing consumer preferences.

In sum, explicitly modeling consumer preferences over product variety and utilizing both the intertemporal investment and intratemporal production Euler conditions are together required for enhancing the estimation precision and economic appeal of consumer preference parameters. We also show that adding lagged industry and market returns as IVs further improves estimation efficiency. Our results complement findings in other areas of finance where richer parameterization of consumer preferences and using novel data improves identification. For example, Huang and Shaliastovich (2014) show that using recursive preferences—in particular, imposing parameteric restrictions for early resolution of uncertainty—and using equity options data helps identification of objective probabilities and risk adjustments.

6.2 Effects of industry production data in IVs

We examine next the effects using industry production data in IVs. To do so, we replace the production-data-based IVs in Table 4 with aggregate IVs utilizing aggregate per capita consumption growth. That is, in place of industry-level investment and materials expenditures data that we utilized in IVs in the estimation in Table 4, we use consumption growth. The lag structure for consumption growth in the IVs is the same as the lag structure for industry data in the original (Table 4) IVs. We label the new IVs as $\Pi_j', j = 1, ...5$, for convenience.

The results are displayed in Table 6. Comparing the results in Table 6 with those in Table 4, we find that the estimates for risk aversion and IES are no longer statistically significant. While the risk aversion levels are reasonable, the estimates for IES are not appealing, essentially being 0. On the other hand, the estimates of ES, σ, are essentially the same as in Table 4 and statistically significant. We also note that the p-value for the J statistics are larger for each specification in Table 6 compared with the corresponding specification in Table 4. Overall, we conclude that IVs using industry production data significantly improve estimation efficiency relative to IVs using aggregate consumption data.

We conclude that incorporating consumer preferences over product variety—parameterized in our model by the ES (σ)—adds structural restrictions to the canonical asset pricing model and helps explain the strong identification of risk aversion and IES, along with the ES.

7 Summary and conclusions

Economic agents generally choose consumption bundles or baskets of various types of consumption goods produced by firms in different sectors or industries. This study examines the real and financial effects of consumer preferences for variety in a dynamic multisector general equilibrium model where the representative consumer is endowed with recursive CES preferences on baskets of goods and firms choose material inputs and capital investment. In addition, we utilize structural restrictions on the representative consumer's consumption-portfolio investment and firms' capital investment and input policies to form moment conditions in GMM estimation of salient parameters that govern recursive consumer preferences in production economies with consumption variety: risk aversion, intertemporal elasticity of substitution, and the product elasticity of substitution (ES).

We find a significant negative relation of ES and equity risk premiums, which we establish through a quantitative analysis of industry equilibrium paths and a cross-sectional analysis of stock returns using a plausible measure of industry ES. We provide intuition for this result by showing that firms' optimal capital investment and output policies in high product substitutability (ES) industries are more sensitive to aggregate shocks compared with firms in low ES industries. This facilitates greater income smoothing for shareholders in high ES industries, resulting in ceteris paribus lower equity risk premiums compared with low ES industries. Our analysis helps reconcile and resolve conflicting evidence in the literature on the relation of industry-level equity risk premiums and product substitutability.

We also find that industry investment and production data provide strong instruments due to high autocorrelations. We obtain economically appealing, efficient estimates of risk-aversion, IES, and ES. Incorporating preferences over product variety in the canonical asset pricing model thus adds structural restrictions that enhance identification of salient parameters governing recursive preferences over consumption of product baskets. Identification diagnostics indicate that Euler conditions from the industry production equilibrium and industry production-based IVs are critical for efficient estimation. In particular, the intertemporal investment Euler condition contributes significantly to the identification of risk aversion and IES, whereas the intratemporal materials optimality condition contributes significantly to the identification of ES. Consequently, both the real restrictions contribute to the identification of risk aversion, IES, and ES.

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Table 1. Calibration for Quantitative Analysis

	Aggregate Parameters							Sectoral Parameters			
Parameter	Φ_w	Φ_π	$\Phi_{w\pi}$	ρ_w	ρ_π	α	γ	η^{-1}	υ	δ	σ
	2.47%	0.76%	4.8e-05	0.95	0.965	0.97	5	0.19	0.1	0.2	2.03
	Sectoral Parameters										
Parameter	ψ_K	ψ_H	Φ_{aw}	$\Phi_{a\pi}$	Φ_{xw}	$\Phi_{x\pi}$	Φ_{zw}	$\Phi_{z\pi}$	ρ_a	ρ_x	ρ_z
	0.21	0.79	4e-05	4.2e-05	3.9e-04	5.5e-05	3e-04	-1.2e-07	0.930	0.966	0.962

This table displays the parameterization used for the numerical computations of the model equilibrium presented in Section 3. ρ_w, ρ_π are the estimated autocorrelation coefficients of the first order autoregressive processes of annual log per capita U.S. income ($w_t = \log W_t$) and log price index ($\pi_t = \log\left(W_t/C_t\right)$, $C_t =$ annual per capita U.S. consumption), that is, $w_t = \rho_w w_{t-1} + \varepsilon_{wt}$ and $\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_{\pi t}$. Φ_w, Φ_π and $\Phi_{w\pi}$ are the volatilities and covariance of the estimated income and price index shocks ε_{wt} and $\varepsilon_{\pi t}$. The intertemporal elasticity of substitution (η^{-1}) and intratemporal product elasticity of substitution are calibrated from the structural estimation of the model (in Table 3). The production and sectoral parameters are calibrated for the U.S. manufacturing sector for 1958-2016 using the NBER-CES (annual) data. The output elasticities of capital and material inputs ψ_K, ψ_H are based on estimation of the Cobb-Douglas production function in (8), while the autocorrelation coefficients of log of productivity shocks and logs of materials price index and investment price index (ρ_a, ρ_x, ρ_z), as well as the covariances of industry productivity and input price shocks with w_t and π_t ($\Phi_{aw}, \Phi_{a\pi}, \Phi_{xw}, \Phi_{x\pi}, \Phi_{zw}, \Phi_{z\pi}$) are estimated from the first order autoregressive processes of log productivity and material inputs during 1958-2016.

Table 2. Product Similarity and Stock Returns

Intercept	β	ln(ME)	ln(BE/ME)	Prod. Similarity
1.205***				-4.582
(3.584)				(-1.570)
1.220***	-0.019			-4.841*
(4.109)	(-0.243)			(-1.749)
1.475	-0.029	-0.015		-4.724*
(1.474)	(-0.344)	(-0.367)		(-1.688)
1.220	-0.015	0.005	0.172**	-5.592**
(1.159)	(-0.178)	(0.105)	(2.113)	(-2.163)

This table analyzes the effect of product similarity (Prod. Similarity) on stock returns using the predictive cross-sectional Fama-MacBeth regressions. The regressions are estimated each month and the reported coefficients are the time-series mean of the estimates. The number in the parentheses reports the Newey-West adjusted t-statistic. The sample period is from 1988 to 2023. β indicates CAPM beta; ME indicates market value of equity; and BE indicates the book value of equity. The returns are measured in percentages. Stars indicate significance at the 10% (*), 5% (**), and 1% (***) levels.

Table 3. Matrix of Autocorrelation Coefficients

Variables	I_t	I_{t-1}	I_{t-2}	H_t	H_{t-1}	H_{t-2}
I_t	1.000					
I_{t-1}	0.971	1.000				
I_{t-2}	0.934	0.972	1.000			
H_t	0.972	0.955	0.941	1.000		
H_{t-1}	0.964	0.973	0.984	0.990	1.000	
H_{t-2}	0.943	0.966	0.973	0.979	0.990	1.000

This table uses annual data from 1958 to 2016. It presents own and cross-autocorrelations of mean annual industry investment (I), materials input (H), and average productivity (A) using the NBER-CES manufacturing database.

Table 4. GMM Estimation of Consumer Preference Parameters

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
Π_1	0.879 (5)***	0.150	0.19***	0.626	2.03***	0.022	8.155	13	0.833
Π_2	0.865 (5)***	0.185	0.19***	0.992	2.02***	0.023	6.049	11	0.870
Π_3	0.864 (6)***	0.188	0.19***	1.1540	2.02***	0.023	6.048	10	0.811
Π_4	0.861 (5)***	0.190	0.19***	1.059	2.02***	0.030	5.979	10	0.817
Π_5	0.871 (5)***	0.182	0.19***	0.922	2.02***	0.021	6.074	12	0.912

This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The moment restrictions are derived from the Euler conditions for capital investment and materials input, as well as the asset markets considered. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The sample period is 1958-2016 (annual) and data sources are described in text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,**, and ***, respectively.

Table 5. Role of Capital Investment and Production Moment Restrictions

Euler Equations	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
All, $\sigma = 2.03$	0.888 (5)***	0.129	0.19***	0.515			5.707	14	0.973
Investment, Asset Markets, $\sigma = 2.03$	0.847 (5)***	0.222	0.19***	1.5442			7.284	10	0.698
Material Inputs, Asset Markets	5.817 (836.2)*	3.049	0.01***	29.817	2.04***	0.044	2.892	3	0.409
Investment, Material Inputs	0.887 (4)*	0.146	0.23**	0.529	2.05***	0.043	5.482	5	0.360

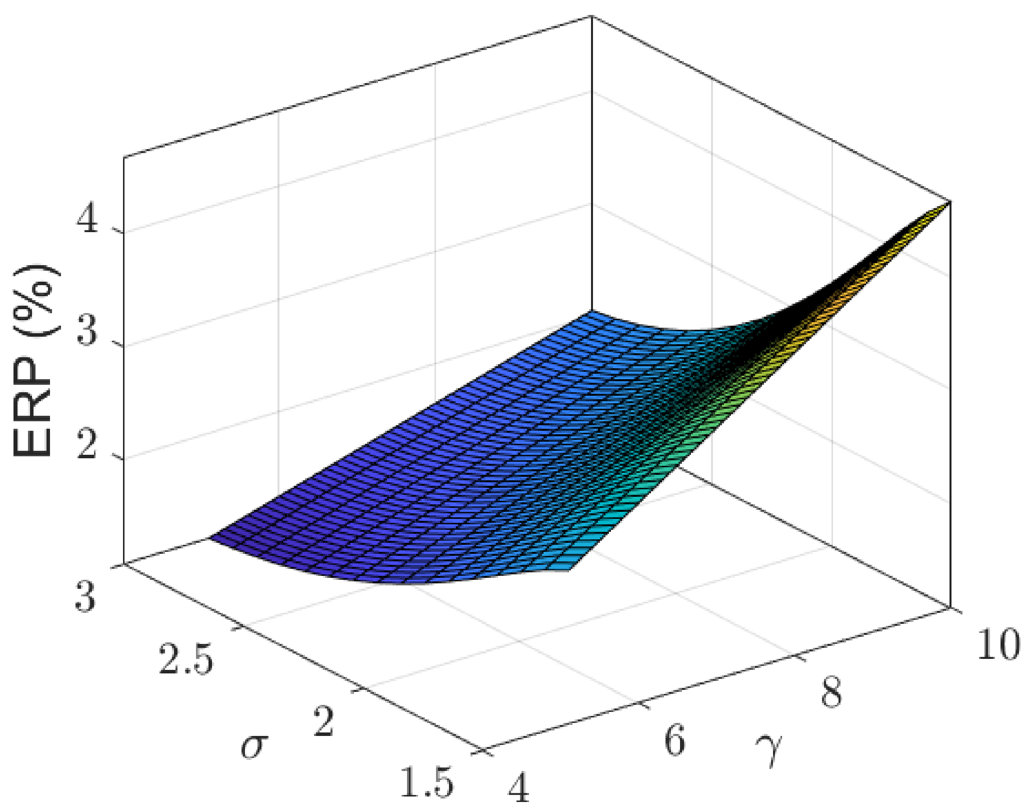
This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The combinations of moment restrictions are derived from the Euler conditions for capital investment and materials inputs, and asset markets, as considered in Table 3. We use the instrumental variables specified in Π_1 in Table 4. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 6. Role of Capital Investment and Production IVs

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
Π'_1	0.003 (4)	0.189	0.001	5.949e+4	1.97***	0.022	4.506	13	0.984
Π'_2	0.001 (3)	0.305	0.00	5.064e+5	1.96***	0.0214	4.473	10	0.924
Π'_3	0.001 (3)	0.204	0.00	2.225e+5	1.97***	0.022	4.408	10	0.927
Π'_4	0.002 (3)	0.182	0.001	1.109e+5	1.97***	0.020	4.739	10	0.907
Π'_5	0.003 (3)	0.188	0.001	4.257e+4	1.96***	0.013	4.566	12	0.971

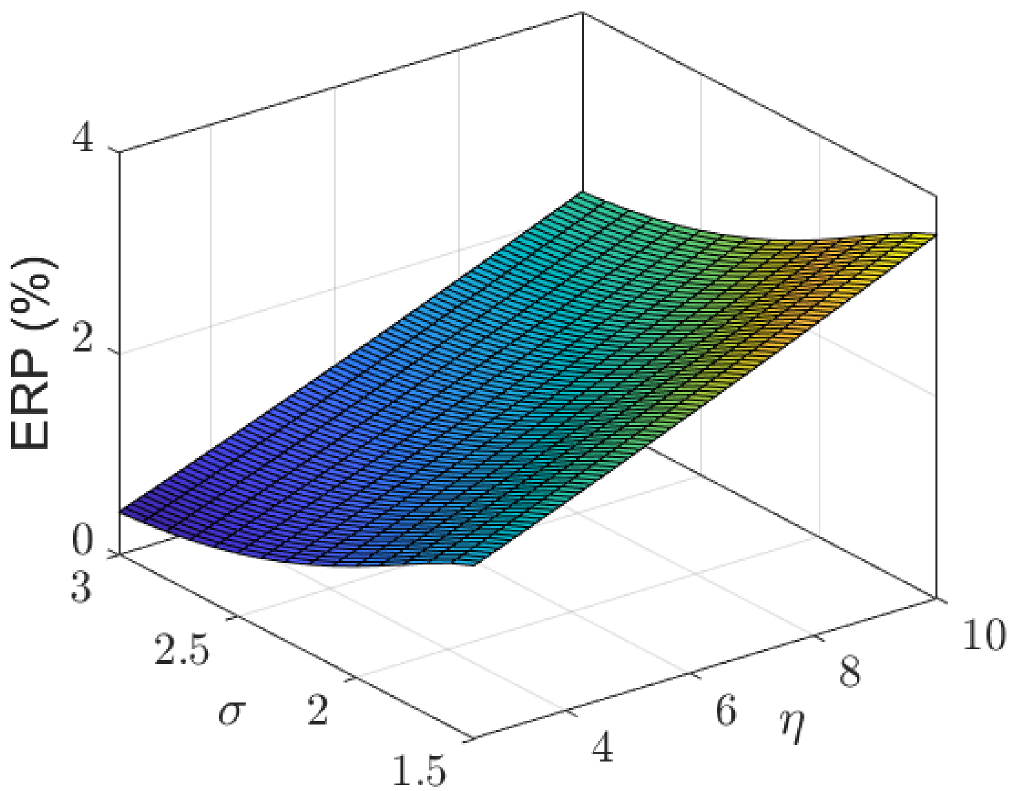
This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The combinations of moment restrictions are derived from the Euler conditions for capital investment and materials inputs, and asset markets, as considered in Table 4. To construct instrumental variables in this table, we start with instrumental variables $\Pi_1 - \Pi_5$ in Table 4 and replace industry production variables with aggregate consumption growth resulting in $(\Pi'_1 - \Pi'_5)$. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Figure 1: Product elasticity of substitution (σ), risk aversion (γ), and equity risk premium



Notes to Figure: This figure graphically displays, through three-dimensional plots, the relation of equilibrium equity risk premium for the consumer durables manufacturing industry, with various combinations of consumer preference parameters: risk aversion (γ), (inverse of) intertemporal elasticity of substitution (η) and intratemporal elasticity of substitution (σ). The sample period is 1958-2016 (annual) and data sources are described in text.

Figure 2: Product elasticity of substitution (σ), intertemporal elasticity of substitution (η^{-1}), and equity risk premium



Notes to Figure: This figure graphically displays, through three-dimensional plots, the relation of equilibrium equity risk premium for the consumer durables manufacturing industry, with various combinations of consumer preference parameters: risk aversion (γ), (inverse of) intertemporal elasticity of substitution (η) and intratemporal elasticity of substitution (σ). The sample period is 1958-2016 (annual) and data sources are described in text.

Internet appendix

A.1 Derivation of optimal consumption and portfolio policies

The representative consumer-investor's (CI's) optimization problem at any t is to

$$\max_{\mathbf{c}_t,\mathbf{q}_{t+1}} \mathcal{U}_t, \text{ s.t.,} \tag{A1.1}$$

$$\mathbf{p}_t \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t. \tag{A1.2}$$

The Lagrangian for with respect to (A1.1)-(A1.2) is

$$\max_{\mathbf{c}_t,\mathbf{q}_{t+1}} \mathcal{U}_t + \chi_t \left[W_t - \mathbf{p}_t \cdot \mathbf{c}_t \right], \; W_t = \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t, \tag{A1.3}$$

where Lagrange multiplier for the budget constraint $\chi_t > 0$ since preferences are strictly increasing in consumption and the budget constraint (A1.2) will be binding in optimum. Using concavity of the objective and convexity of the constraint, the optimal consumption and portfolio policies can be characterized through a two-step process, where the optimal consumption vector $\mathbf{c}_t$ is first determined as a function of available consumption expenditure W_t, and the portfolio $\mathbf{q}_{t+1}$ is then determined taking as given the optimal consumption policy.

Then, using the definition of the consumption basket C_t in (2), the first order optimality conditions for $c_{jt}, j = 1, ...J$, can be written

$$[(1-\alpha)(1-\eta)](C_t)^{\frac{1-\eta\sigma}{\sigma}}(c_{jt})^{-\frac{1}{\sigma}}\phi_j = \chi_t p_{jt}. \tag{A1.4}$$

Isolating c_{jt} in (A1.4) and multiplying both sides by p_{jt} yields

$$p_{jt}c_{jt} = \chi_t^{-\sigma}(p_{jt})^{1-\sigma}(C_t)^{-(1-\eta\sigma)}(\phi_j)^{\sigma}[(1-\alpha)(1-\eta)]^{\sigma}. \tag{A1.5}$$

Then recognizing that $W_t = \sum_j p_{jt}c_{jt}$ and $P_t = \left[\sum_{j=1}^{J} (\phi_j)^{\sigma}(p_{jt})^{1-\sigma} \right]^{1/(1-\sigma)}$, summing both sides of (A1.5) over j allows one to solve for the Lagrange multiplier as

$$\chi_t = \left(\frac{W_t}{P_t} \right)^{-\frac{1}{\sigma}} P_t^{-1}(C_t)^{\frac{1-\eta\sigma}{\sigma}}[(1-\alpha)(1-\eta)]. \tag{A1.6}$$

Substituting for χ_t in (A1.4) and rearranging terms then gives the optimal consumption functions in (4), that is,

$$c_{jt}(\mathbf{p}_t^c, W_t) = \frac{W_t}{P_t} \left[\frac{P_t \phi_j}{p_{jt}} \right]^{\sigma}, \; j = 1, ..J. \tag{A1.7}$$

Now (A1.7) implies

$$(C_t)^{\frac{\sigma-1}{\sigma}} = \sum_j \phi_j (c_{jt})^{\frac{\sigma-1}{\sigma}} = \left(\frac{W_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}} (P_t)^{\sigma-1} \left(\sum_j (\phi_j)^\sigma (p_{jt})^{1-\sigma}\right). \tag{A1.8}$$

But since $\sum_j (\phi_j)^\sigma (p_{jt})^{1-\sigma} = (P_t)^{1-\sigma}$, (A1.8) yields $C_t = \frac{W_t}{P_t}$.

Next, conditional on optimal $\mathbf{c}_t$ and, hence, $C_t = \frac{W_t}{P_t}$, the derivation of the optimal portfolio condition (6) with Epstein and Zin (1989) preferences is standard using straightforward application of arguments in Epstein and Zin (1989). ∎

A.2 Derivation of optimal intermediate goods policies

We derive the optimal demand for material intermediate goods. The derivation for optimal investment intermediate goods is analogous.

The firm's objective is to maximize effective materials input

$$H_t^j = \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{nt}^j)^{(\zeta_j^h-1)/\zeta_j^h}\right]^{\zeta_j^h/(\zeta_j^h-1)} \tag{A2.1}$$

subject to a fixed materials cost Υ_{ht}^j. Hence, the Lagrangian is

$$\max_{H_{nt}^j} \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{nt}^j)^{(\zeta_j^h-1)/\zeta_j^h}\right]^{\zeta_j^h/(\zeta_j^h-1)} + \chi_{ht}^j \left[\Upsilon_{ht}^j - \sum_{n=J_c+1}^{J_h} p_t^n H_{nt}^j\right]. \tag{A2.2}$$

This yields the optimality conditions for $H_{nt}^j, n = 1, ...J$

$$(H_t^j)^{\frac{1}{(\zeta_j^h-1)}} (H_{nt}^j)^{-\frac{1}{\zeta_j^h}} \varphi_{nj} = \chi_{ht}^j p_t^n, \tag{A2.3}$$

which implies that

$$p_t^n H_{nt}^j = (\chi_{ht}^j)^{-\zeta_j^h} (H_t^j)^{\frac{\zeta_j^h}{(\zeta_j^h-1)}} (p_t^n)^{1-\zeta_j^h} (\varphi_{nj})^{\zeta_j^h}. \tag{A2.4}$$

Summing both sides of (A2.4) over n gives

$$\chi_{ht}^j = (\Upsilon_{ht}^j)^{-\frac{1}{\zeta_j^h}} (H_t^j)^{\frac{1}{(\zeta_j^h-1)}} \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{1/\zeta_j^h}. \tag{A2.5}$$

Substituting (A2.5) in (A2.3) yields,

$$H_{nt}^j = (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{-\zeta_j^h} \Upsilon_{ht}^j \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1}. \tag{A2.6}$$

Now put $X_t^j = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{1/(1-\zeta_j^h)}$ so that $X_t^j = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1} = (X_t^j)^{\zeta_j^h - 1}$. Hence,

$$\Upsilon_{ht}^j \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1} = \left(\frac{\Upsilon_{ht}^j}{X_t^j}\right) (X_t^j)^{\zeta_j^h}. \tag{A2.7}$$

Let us now conjecture that $\left(\frac{\Upsilon_{ht}^j}{X_t^j}\right) = H_t^j$, so that (A2.6)-(A2.7) together yield,

$$H_{nt}^j = (\varphi_{nj}^h)^{\zeta_j^h} \left[\frac{p_t^n}{X_t^j}\right]^{-\zeta_j^h} H_t^j. \tag{A2.8}$$

Then using (A2.8) in $\Upsilon_{ht}^j = \sum_{n=J_c+1}^{J_h} p_t^n H_{nt}^j$ indeed verifies that $\frac{\Upsilon_{ht}^j}{X_t^j} = H_t^j$. ∎

A.3 Derivation of equilibrium conditions

As in the text, we suppress the notation for sectors (unless necessary) for expositional ease. Using the Bellman-representation (16), along any competitive equilibrium path at any t, conditional on $\Omega_t = (W_t, P_t, A_t, X_t, Z_t, K_t)$, the optimization problem for the typical competitive firm is

$$V_t(\Omega_t) = \max_{I_t, H_t \geq 0} D_t + \mathbb{E}_t \left[V_{t+1}(\Omega_{t+1})\right], \text{ s.t.,} \tag{A3.1}$$

$$D_t = p_t Y_t - X_t H_t - O(I_t, K_t), \tag{A3.2}$$

$$p_t = \phi \left(W_t P_t^{\sigma-1}\right)^{1/\sigma} (Y_t)^{-1/\sigma}, \tag{A3.3}$$

$$Y_t = F(K_t, H_t, A_t) = A_t (K_t)^{\psi_K} (H_t)^{\psi_H}, \tag{A3.4}$$

$$O(I_t, K_t) = Z_t I_t + 0.5 \upsilon \left(\frac{I_t}{K_t}\right)^2 K_t, \tag{A3.5}$$

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{A3.6}$$

Taking the equilibrium sectoral price p_t as given, optimization with respect to H_t then yields

$$p_t F_H(K_t, H_t, A_t) = X_t. \tag{A3.7}$$

Next, the optimal (interior) I_t satisfies

$$0 = \frac{\partial D_t}{\partial I_t} + \mathbb{E}_t \left[\frac{\partial V_{t+1}(\Omega_{t+1})}{\partial K_{t+1}}\right]. \tag{A3.8}$$

But from (A3.2) and (A3.5),

$$\frac{\partial D_t}{\partial I_t} = -O_I(I_t, K_t) = -\left[Z_t + \upsilon \left(\frac{I_t}{K_t}\right)\right]. \tag{A3.9}$$

And using the intertemporal envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen I_{t+1} and H_{t+1} to zero), along the competitive equilibrium path

$$\begin{aligned}\frac{\partial V_{t+1}(\Omega_{t+1})}{\partial K_{t+1}} &= p_{t+1}F_K(K_{t+1},H_{t+1},A_{t+1}) - O_K(I_{t+1},K_{t+1}) - (1-\delta)\frac{\partial D_{t+1}}{\partial I_{t+1}} \\ O_K(I_{t+1},K_{t+1}) &= -0.5\upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)^2, \frac{\partial D_{t+1}}{\partial I_{t+1}} = -O_I(I_{t+1},K_{t+1}). \qquad \text{(A3.10)}\end{aligned}$$

(A3.8) then yields,

$$\begin{aligned}\left[Z_t + \upsilon\left(\frac{I_t}{K_t}\right)\right] &= \mathbb{E}_t\left[\Omega_{t,t+1}\left\{p_{t+1}F_K(K_{t+1},H_{t+1},A_{t+1}) + 0.5\upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \right.\right. \\ &\qquad \left.\left.(1-\delta)\left[Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right]\right\}\right], \qquad \text{(A3.11)}\end{aligned}$$

where $K_{t+1} = K_t(1-\delta) + I_t$. The equilibrium product price is then obtained from (A3.3) as

$$p_t = \phi\left(\frac{W_t}{P_t}\right)^{1/\sigma} P_t\left[F(K_t,H_t,A_t)\right]^{-1/\sigma}. \qquad \text{(A3.12)}$$

Finally, the equilibrium asset market restriction in Equation (21) follows from Equation (7). ∎

B. Equilibrium computations

B.1 Steady state computations

We will denote the deterministic steady state with bars. Note that in the deterministic steady state the pricing kernel is $\bar{\Lambda} = \alpha$ because $W_{t+1} = W_t = \bar{W}, P_{t+1} = P_t = \bar{P}$, and $\mathcal{U}_t = \mathcal{U}_{t+1}$. Now, in the deterministic steady state, the equilibrium input choice and and capital stock $(\bar{H},\bar{K})$ are fixed (and hence $\bar{I} = \delta\bar{K}$). These quantities are derived from the optimality conditions (18) and (19) in the text. In the steady state, the Euler conditions are

$$\begin{aligned}\bar{p}F(\bar{K},\bar{H},\bar{A}) &= \bar{X}, \qquad \text{(B.1.1)} \\ -(\bar{Z}+\upsilon\delta) + \alpha\left[\bar{p}F(\bar{K},\bar{H},\bar{A}) + 0.5\upsilon(\delta)^2 + (1-\delta)(\bar{Z}+\upsilon\delta)\right] &= 0, \qquad \text{(B.1.2)}\end{aligned}$$

where the steady state sector prices and sales are

$$\begin{aligned}\bar{p} &= \phi\bar{W}^{1/\sigma}\bar{P}^{\frac{\sigma-1}{\sigma}}[F(\bar{K},\bar{H},\bar{A})]^{-1/\sigma}, \qquad \text{(B.1.3)} \\ \bar{\Psi} &= \phi\bar{W}^{1/\sigma}\bar{P}^{\frac{\sigma-1}{\sigma}}[F(\bar{K},\bar{H},\bar{A})]^{(\sigma-1)/\sigma} \qquad \text{(B.1.4)}\end{aligned}$$

Noting that $F(\bar{K},\bar{H},\bar{A}) = \bar{A}(\bar{K})^{\psi}(\bar{H})^{\psi}$, $F(\bar{K},\bar{H},\bar{A}) = \psi F(\bar{K},\bar{H},\bar{A})/H$, $F(\bar{K},\bar{H},\bar{A}) = \psi F(\bar{K},\bar{H},\bar{A})/K$, and substituting (B.1.3)-(B.1.4) in (B.1.1)-(B.1.2) yields the system of equations

$$\bar{H} = \left[(\phi\psi)^{\sigma}\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1}(\bar{K})^{(\sigma-1)\psi}(\bar{X})^{-\sigma}\right]^{1/\nu}, \tag{B.15}$$

$$\bar{K} = \left[\left(\frac{\alpha\phi\psi}{e}\right)^{\sigma}\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1}(\bar{H})^{(\sigma-1)\psi}\right]^{1/\nu}, \tag{B.1.6}$$

where $\nu \equiv \psi + \sigma(1-\psi)$, $\nu \equiv \psi + \sigma(1-\psi)$, and $e \equiv \bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2 - \alpha(2+\delta))$.

To solve for $(\bar{H}, \bar{K})$ analytically, it is convenient to take the log of both sides of (B.15)-(B.1.6). Using small letters to express logs, setting $\bar{\pi} = \log(\bar{P})$), and solving for $\bar{H}$we get

$$\begin{aligned} \bar{H} &= \exp\left((\varsigma)^{-1}\left[\sigma(\log\phi + \log\psi - \bar{x}) + (\sigma-1)\left(\frac{\psi}{\nu}\right)\times\right.\right. \\ &\quad \left.\left.(\log\alpha + \log\phi + \log\psi - \log e) + \left(1 + (\sigma-1)\left(\frac{\psi}{\nu}\right)\right)[\bar{w} + (\sigma-1)(\bar{a}+\bar{\pi})]\right]\right) \\ &= \left[(\phi\psi)^{\sigma}(\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1})^{n}\left(\frac{\alpha\phi\psi}{e}\right)^{\varsigma}(\bar{X})^{-\sigma}\right]^{1/o}, \end{aligned} \tag{B.1.7}$$

where $\varsigma \equiv (\nu x - (\sigma-1)^2\psi(\psi/\nu))$, $\varsigma \equiv (\sigma-1)(\psi/\nu)$, and $n \equiv 1 + \varsigma$. $\bar{K}$is then recovered from substituting (B.1.7) in (B.1.6).

B.2 Analytic approximations

B.2.1 Pricing kernel

From (5), the log of the real pricing kernel $\lambda_{t+1} \equiv \log(\Lambda_{t,t+1})$ is

$$\lambda_{t+1} = \theta\log\alpha - \eta\theta\left[g_{w,t+1} - g_{\pi,t+1}\right] + (\theta-1)r_{c,t+1}, \tag{B.2.1}$$

where $g_{w,t+1}$ and $g_{\pi,t+1}$ are the log growth rates of income and aggregate price, respectively, and $r_{c,t+1} \equiv \log(R_{C,t+1})$. Using the Campbell and Shiller (1988) log-linearization approach, we can represent $r_{c,t+1}$ as

$$r_{c,t+1} = f_0 + f_1 z_{c,t+1} - z_{ct} + g_{w,t+1} - g_{\pi,t+1}, \tag{B.2.2}$$

where z_{ct} is the log price-consumption ratio. Here f_0 and f_1 are approximating constants that depend on the unconditional mean of z_{ct}, say, z_c. (In our numerical computations, we take z_c to be the mean of simulated $(w_t - p_t)$.) Indeed,

$$f_0 = \log(1 + \exp(z_c)) - f_1 z_c;\ f_1 = \frac{\exp(z_c)}{1+\exp(z_c)}. \tag{B.2.3}$$

Exploiting the (log form of) the Euler condition (5), in the setting at hand z_{ct} is a linear function of the logs of the aggregate state variables W_t and P_t, namely,

$$z_{ct} = \kappa_{c0} + \kappa_{cw} w_t + \kappa_{c\pi} \pi_t. \tag{B.2.4}$$

Since $g_{w,t+1} = (\rho_w - 1)w_t + \varepsilon^w_{t+1}$ and $g_{\pi,t+1} = (\rho_\pi - 1)\pi_t + \varepsilon^\pi_{t+1}$, substitution of (B.2.2) and (B.2.4) in (B.2.1) allows one to write

$$\lambda_{t+1} = B_0 + B_w w_t + B_\pi \pi_t + b_w \varepsilon_{w,t+1} + b_\pi \varepsilon_{\pi,t+1}, \tag{B.2.5}$$

where

$$\begin{aligned}
B_0 &= \theta \log \alpha; \\
B_w &\equiv (\rho_w - 1)\left[1 - \eta\theta + (\theta - 1) f_1 \kappa_{cw}\right] - (\theta - 1)\kappa_{cw}; \\
B_\pi &\equiv (\rho_\pi - 1)\left[\eta\theta - 1 - (\theta - 1) f_1 \kappa_{c\pi}\right] + (\theta - 1)\kappa_{c\pi}; \\
b_w &\equiv -\eta\theta + (\theta - 1)[f_1 \kappa_{cw} + 1]; \\
b_\pi &\equiv \eta\theta - (\theta - 1)[f_1 \kappa_{c\pi} + 1].
\end{aligned} \tag{B.2.6}$$

We can then obtain the coefficients of z_{ct} in (B.2.4) through the method of undetermined coefficients. The Euler condition (5) is

$$\begin{aligned}
1 &= \mathbb{E}_t[\exp(\lambda_{t+1} + \pi_t - \pi_{t+1} + r_{c,t+1})] \\
&= \mathbb{E}_t[\exp(\theta \log \alpha - \eta\theta \left[g_{w,t+1} - g_{\pi,t+1}\right] + \theta r_{c,t+1} + \pi_t - \pi_{t+1})]
\end{aligned} \tag{B.2.7}$$

Since (B.2.7) must hold for all values of the state variables, all terms involving w_t and π_t must satisfy

$$w_t \theta[(\rho_w - 1)(1 - \eta) + \kappa_{cw}(f_1(\rho_w - 1) - 1)] = 0, \tag{B.2.8}$$

$$\pi_t \left\{\theta[(\rho_\pi - 1)(\eta - 1) + \kappa_{c\pi}(f_1(\rho_\pi - 1) - 1)] + 1 - \rho_\pi\right\} = 0. \tag{B.2.9}$$

From (B.2.8)-(B.2.9), it follows

$$\kappa_{cw} = \frac{(\rho_w - 1)(\eta - 1)}{f_1(\rho_w - 1) - 1}, \kappa_{c\pi} = \frac{(\rho_\pi - 1)[\theta(1 - \eta) + 1]}{\theta(f_1(\rho_\pi - 1) - 1)}. \tag{B.2.10}$$

And to ensure that the constant terms in (B.2.7) equal zero, from (B.2.2) and (B.2.4), the coefficient κ_{c0} is calculated as, $\kappa_{c0} = \frac{\log \alpha + f_0}{1 - f_1}$. Finally, note that the log of the nominal pricing kernel $m_{t+1} = \lambda_{t+1} + \pi_t - \pi_{t+1}$ is

$$m_{t+1} = B_0 + B_w w_t + (B_\pi + (1 - \rho_\pi))\pi_t + b_w \varepsilon_{w,t+1} + b_\pi \varepsilon_{\pi,t+1}. \tag{B.2.11}$$

B.2.2 Approximations of industry equilibrium conditions

Note that the optimality condition for materials inputs (18) can be written

$$
\begin{aligned}
0 &= \frac{\psi_H \Psi_t}{H_t} - X_t, \text{ where} \\
\Psi_t &= \phi W_t^{1/\sigma} P_t^{\frac{\sigma-1}{\sigma}} \left[A_t (K_t)^{\psi_K} (H_t)^{\psi_H}\right]^{\frac{(\sigma-1)}{\sigma}} \qquad \text{(B.2.12)}
\end{aligned}
$$

Hence, log-linearization of the optimality condition around the steady state implies

$$\left(\frac{\psi_H \bar{\Psi}}{\bar{H}}\right) \left[\sigma + \hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \nu_H \hat{H}_t\right] - \sigma \bar{X}(1 + \hat{X}_t) = 0, \qquad \text{(B.2.13)}$$

where $\nu_H \equiv \psi_H + \sigma(1 - \psi_H)$ has been defined above. But using the fact that in the steady state $\psi_H \bar{\Psi}(\bar{H})^{-1} = \bar{X}$, $(B.2.13)$ gives

$$\left(\frac{\psi_H \bar{\Psi}}{\bar{H}}\right) \left[\hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \nu_H \hat{H}_t\right] - \sigma \bar{X} \hat{X}_t = 0 \qquad \text{(B.2.14)}$$

Dividing through by $\psi_H \bar{\Psi}(\bar{H})^{-1}$ and noting that $\bar{X}/(\psi_H \bar{\Psi}(\bar{H})^{-1}) = 1$ and rearranging terms then yields

$$\hat{H}_t = (\nu_H)^{-1} \left[\hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \sigma \hat{X}_t\right]. \qquad \text{(B.2.15)}$$

To derive the log-linearized form of equilibrium investment, we use the fact that

$$I_t = K_{t+1} - (1 - \delta) K_t$$

and reformulate the optimization problem for investment (in the standard way) as the choice of K_{t+1} at t. Noting that

$$\frac{\partial \Psi_{t+1}}{\partial K_{t+1}} = p_{t+1} F_K(K_{t+1}, H_{t+1}, A_{t+1}) = \psi_K \frac{\Psi_{t+1}}{K_{t+1}}, \qquad \text{(B.2.16)}$$

the investment Euler condition (19) can be written

$$Z_t + \upsilon\left(\frac{I_t}{K_t}\right) = \mathbb{E}_t \left[M_{t,t+1} \left\{\psi_K \frac{\Psi_{t+1}}{K_{t+1}} + \left(\frac{\upsilon}{2}\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1 - \delta)\left(Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right\}\right]. \qquad \text{(B.2.17)}$$

To derive the equilibrium law of motion for capital stock, one looks for a second order difference equation in $\hat{K}_t$. We will log-linearize (B.2.17) and then use the log-linearization of the investment transition equation to get:

$$\hat{I}_t = (\delta)^{-1} \left(\hat{K}_{t+1} - \hat{K}_t (1 - \delta)\right). \qquad \text{(B.2.18)}$$

In (B.2.18), we use the fact that $\bar{I} = \delta \bar{K}$, so that $\bar{K}/\bar{I} = 1/\delta$. Then log-linearization of

(B.2.17) implies (noting that $\bar{M} = \alpha$) :

$$\begin{aligned}\bar{Z}(1+\hat{Z}_t) + \upsilon\delta(1+\hat{I}_t - \hat{K}_t) \quad &= \quad \alpha\mathbb{E}_t\Bigg[\psi_K \frac{\bar{\Psi}}{\bar{K}}(1+\hat{M}_{t,t+1} + \hat{\Psi}_{t+1} - \hat{K}_{t+1}) + (1-\delta)\bar{Z}\times \\ &(1+\hat{Z}_{t+1} + \hat{M}_{t,t+1}) \ + \left(\frac{\upsilon}{2}\right)(\delta)^2(1+\hat{M}_{t,t+1} + 2(\hat{I}_{t+1} - \hat{K}_{t+1})) \ + \\ &\upsilon\delta(1-\delta)(1+\hat{M}_{t,t+1} + (\hat{I}_{t+1} - \hat{K}_{t+1}))\Bigg] \qquad \text{(B.2.19)}\end{aligned}$$

Rearranging terms, the RHS of (B.2.19) can be written

$$\begin{aligned}&\alpha\mathbb{E}_t\Bigg[(1+\hat{M}_{t,t+1})\left(\psi_K\frac{\bar{\Psi}}{\bar{K}} + (1-\delta)\bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2-\delta)\right) + \\ &(1-\delta)\bar{Z}\hat{Z}_{t+1} + \psi_K\frac{\bar{\Psi}}{\bar{K}}(\hat{\Psi}_{t+1} - \hat{K}_{t+1}) + \upsilon\delta(\hat{I}_{t+1} - \hat{K}_{t+1})\Bigg]. \qquad \text{(B.2.20)}\end{aligned}$$

But in the steady state, the Euler for investment is

$$Z + \upsilon\delta = \alpha\left[\psi_K\frac{\bar{\Psi}}{\bar{K}} + Z(1-\delta) + \left(\frac{\upsilon\delta}{2}\right)(2-\delta)\right]. \qquad \text{(B.2.21)}$$

Combining (B.2.20) and (B.2.21), we can write the log-linearized Euler (B.2.17) as

$$\begin{aligned}0 \quad = \quad &\mathbb{E}_t\Bigg[\hat{M}_{t,t+1}u^M + \psi_K\frac{\bar{\Psi}}{\bar{K}}(\hat{\Psi}_{t+1} - \hat{K}_{t+1}) + \upsilon\delta\left\{(\hat{I}_{t+1} - \hat{K}_{t+1}) - \alpha^{-1}(\hat{I}_t - \hat{K}_t)\right\} + \\ &\bar{Z}\left\{(1-\delta)\hat{Z}_{t+1} - \alpha^{-1}\hat{Z}_t\right\}\Bigg], \qquad \text{(B.2.22)}\end{aligned}$$

where $u^M \equiv \left(\psi_K\frac{\bar{\Psi}}{\bar{K}} + (1-\delta)\bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2-\delta)\right)$. Now note from (B.2.12) that

$$\Psi_{t+1} = \phi W_{t+1}^{1/\sigma} P_{t+1}^{\frac{\sigma-1}{\sigma}}[A_{t+1}(K_{t+1})^{\psi_K}(H_{t+1})^{\psi_H}]^{(\sigma-1)/\sigma}$$

Hence, log-linearization gives

$$\begin{aligned}\hat{\Psi}_{t+1} \quad &= \quad \left(\frac{\phi}{\sigma}\right)\left[\hat{W}_{t+1} + (\sigma-1)\{\hat{P}_{t+1} + \hat{A}_{t+1} + \psi_K\hat{K}_{t+1} + \psi_H\hat{H}_{t+1}\}\right], \qquad \text{(B.2.23)} \\ \hat{H}_{t+1} \quad &= \quad (\nu_H)^{-1}\left[\hat{W}_{t+1} + (\sigma-1)\left\{\hat{P}_{t+1} + \hat{A}_{t+1} + \psi_K\hat{K}_{t+1}\right\} - \sigma\hat{X}_{t+1}\right], \qquad \text{(B.2.24)}\end{aligned}$$

where (B.2.24) follows from (B.2.15). However, from (B.2.18)

$$\hat{I}_{t+i} - \hat{K}_{t+i} = (\delta)^{-1}\left(\hat{K}_{t+i} - \hat{K}_t\right), i = 0, 1$$

Substituting in (B.2.22) then yields

$$0 = \mathbb{E}_t \left[\hat{M}_{t,t+1} u^M + \left(\frac{\phi\psi_K}{\sigma} \right) \frac{\bar{\Psi}}{\bar{K}} \{ (\hat{W}_{t+1} + (\sigma-1)(\hat{P}_{t+1} + \hat{A}_{t+1}))(1+u^H) - u^H \sigma \hat{X}_{t+1} \} + u_2^K \hat{K}_{t+2} + u_1^K \hat{K}_{t+1} + u_0^K \hat{K}_t + \bar{Z} \left\{ (1-\delta)\hat{Z}_{t+1} - \alpha^{-1}\hat{Z}_t \right\} \right] \quad \text{(B.2.25)}$$

where $u^H = (\nu_H)^{-1}\psi_H, u_2^K = \upsilon$, $u_0^K = \left(\frac{\upsilon}{\alpha}\right)$, and

$$u_1^K = \left(\frac{\psi_K \bar{\Psi}}{\bar{K}} \left(\frac{\phi}{\sigma}(\sigma-1)\psi_K(1+u^H) - \sigma \right) - \frac{\upsilon(1+\alpha)}{\alpha} \right). \quad \text{(B.2.26)}$$

We now use the method of undetermined coefficients to ensure that the right hand side of (B.2.25) is zero for all values of the state variables. We will write $k_t = \log(K_t)$. Recall that

$$\hat{M}_{t,t+1} = \lambda_{t+1} + \pi_t - \pi_{t+1} - \log \alpha.$$

Moreover, $\hat{W}_{t+1} = w_{t+1} - \bar{w}$, where $\bar{w} = \log \bar{W}$. Hence, we have

$$\begin{aligned} \hat{W}_{t+1} &= \rho_w w_t + \varepsilon_{t+1}^w - \bar{w}; \\ \hat{P}_{t+1} &= \rho_\pi \pi_t + \varepsilon_{t+1}^\pi - \bar{\pi} \\ \hat{A}_{t+1} &= \rho_a a_t + \varepsilon_{a,t+1} - \bar{a} \\ \hat{X}_{t+1} &= \rho_x x_t + \varepsilon_{x,t+1} - \bar{x}, \\ \hat{Z}_{t+1} &= \rho_z z_t + \varepsilon_{z,t+1} - \bar{z} \end{aligned}$$

and finally $\hat{K}_{t+i} = k_{t+i} - \bar{k}, i = 0, 1, 2$. Then from (B.2.11)

$$\mathbb{E}_t \left[\hat{M}_{t,t+1} u^M \right] = u^M [B_w w_t + (B_\pi \pi_t + (1-\rho_\pi)\pi_t + (\theta-1)\log\alpha] \quad \text{(B.2.27a)}$$

Similarly, the expectation of the second term in (B.2.25) is

$$\left(\frac{\phi\psi_K}{\sigma} \right) \frac{\bar{\Psi}}{\bar{K}} \left\{ \left([\rho_w w_t + (\sigma-1)(\rho_\pi \pi_t + \rho_a a_t)](1+u^H) - u^H \sigma \rho_x x_t \right) - \left[(\bar{w} + (\sigma-1)(\bar{\pi} + \bar{a}))(1+u^H) - u^H \sigma \bar{x} \right] \right\}. \quad \text{(B.2.27b)}$$

Next, we conjecture (and subsequently verify) that k_{t+1} is an affine function of the form

$$k_{t+1} = \xi_0 + \xi_w w_t + \xi_\pi \pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t + \xi_k k_t, \quad \text{(B.2.28)}$$

Hence

$$\mathbb{E}_t \left[u_2^K \hat{K}_{t+2} + u_1^K \hat{K}_{t+1} + u_0^K \hat{K}_t \right] =$$

$$k_t\left[u_2^K(\xi_k)^2+u_1^K\xi_k+u_0^K\right]+w_t\left[\xi_w\{u_2^K(\rho_w+\xi_k)+u_1^K\}\right]+$$
$$\pi_t\left[\xi_\pi\{u_2^K(\rho_\pi+\xi_k)+u_1^K\}\right]+a_t\left[\xi_a\{u_2^K(\rho_a+\xi_k)+u_1^K\}\right]+x_t\left[\xi_x\{u_2^K(\rho_x+\xi_k)+\right.$$
$$\left.u_1^K\}\right]+z_t\left[\xi_z\{u_2^K(\rho_z+\xi_k)+u_1^K\}\right]-\xi_0\left[u_2^K(1+\xi_k)+u_1^K\right]-$$
$$[\xi_w\bar{w}+\xi_\pi\bar{\pi}+\xi_a\bar{a}+\xi_x\bar{x}+\xi_k\bar{k}]. \tag{B.2.29}$$

To ensure that terms multiplying k_t equal zero, the following quadratic equation in ξ_k must be satisfied

$$u_2^K(\xi_k)^2+u_1^K\xi_k+u_0^K=0 \tag{B.2.30}$$

so that

$$\xi_k=-\frac{u_1^K}{2u_2^K}\pm\frac{\sqrt{(u_1^K)^2-4u_2^Ku_0^K}}{2u_2^K} \tag{B.2.31}$$

In the standard way, the root with $\mid\xi_k\mid<1$ will be chosen. Next, collecting terms for w_t in (B.2.27a)-(B.2.29) and requiring them to be zero, we have

$$w_t\left[\xi_w\{u_2^K(\rho_w+\xi_k)+u_1^K\}+u^MB_w+\left(\frac{\phi\psi_K\bar{\Psi}}{\sigma\bar{K}}\right)\rho_w(1+u^H)\right]=0 \tag{B.2.32}$$

so that

$$\xi_w=-\frac{\left(\frac{\phi\psi_K\bar{\Psi}}{\sigma\bar{K}}\right)\rho_w(1+u^H)+u^MB_w}{u_2^K(\rho_w+\xi_k)+u_1^K}, \tag{B.2.33}$$

which is well defined since ξ_k is known from (B.2.31). Similar calculations then show

$$\xi_\pi = -\frac{\left(\frac{\phi\psi_K\bar{\Psi}}{\sigma\bar{K}}\right)(\sigma-1)\rho_\pi(1+u^H)+u^M(B_\pi+(1-\rho_\pi))}{u_2^K(\rho_\pi+\xi_k)+u_1^K}, \tag{B.2.34}$$

$$\xi_a = -\frac{\left(\frac{\phi\psi_K\bar{\Psi}}{\sigma\bar{K}}\right)(\sigma-1)\rho_a(1+u^H)}{u_2^K(\rho_a+\xi_k)+u_1^K}, \tag{B.2.35}$$

$$\xi_x = \frac{\left(\frac{\phi\psi_K\bar{\Psi}}{\bar{K}}\right)u^H\rho_x}{u_2^K(\rho_x+\xi_k)+u_1^K}, \tag{B.2.36}$$

$$\xi_z = \frac{1-\alpha\bar{Z}(1-\delta)\rho_z}{\alpha\left[u_2^K(\rho_z+\xi_k)+u_1^K\right]}. \tag{B.2.37}$$

Finally, to ensure that all constant terms collectively equal zero, we need $\xi_0=T_0/T_1$ where

$$\begin{aligned} T_0 &= [\xi_w\bar{w}+\xi_\pi\bar{\pi}+\xi_a\bar{a}+\xi_x\bar{x}+\xi_z\bar{z}+\xi_k\bar{k}]+\left(\frac{\phi\psi_K\bar{\Psi}}{\sigma\bar{K}}\right)\times \\ &\quad [(\bar{w}+(\sigma-1)(\bar{\pi}+\bar{a}))+(1+u^H)-u^H\sigma\bar{x}]- \\ &\quad u^M(\theta-1)\log\alpha+\upsilon\delta\left(\frac{1+\alpha}{\alpha}\right), \end{aligned} \tag{B.2.38}$$

$$T_1 = u_2^K(1+\xi_k)+u_1^K. \tag{B.2.39}$$

(B.2.31)-(B.2.39) then complete the specification of equilibrium k_{t+1} as a function of current state variables. It is also useful to re-express (B.2.15) in terms of $h_t = \log(H_t)$. Recalling that $\hat{H}_t = h_t - \bar{h}$, (B.2.15) gives

$$h_t - \bar{h} = (\nu_H)^{-1} \left[w_t + (\sigma - 1)(\pi_t + a_t + \psi_K k_t) - \sigma x_t - (\bar{w} + (\sigma - 1)(\bar{\pi} + \bar{a} + \psi_K \bar{k}) - \sigma \bar{x}) \right]. \tag{B.2.40}$$

B.2.3 Equilibrium industry equity risk premium

Log-linearizing dividends in Equation (14) implies

$$\begin{aligned} \bar{D}(1 + \hat{D}_t) &= \bar{\Psi}(1 + \hat{\Psi}_t) - \bar{X}\bar{H}(1 + \hat{X}_t + \hat{H}_t) - [\bar{Z}\bar{I}(1 + \hat{Z}_t + \hat{I}_t) + \\ &\quad 0.5\upsilon \left(\frac{(\bar{I})^2}{\bar{K}} \right) (1 + 2\hat{I}_t - \hat{K}_t)]. \end{aligned} \tag{B.2.41}$$

Using the facts that $\bar{I} = \delta \bar{K}$, $\bar{D} = \bar{\Psi} - \bar{X}\bar{H} - \delta \bar{K}(\bar{Z} + 0.5\upsilon\delta)$, and substituting for $\hat{I}_t$ from (B.2.18) yields

$$\begin{aligned} \bar{D}\hat{D}_t &= \bar{\Psi}\hat{\Psi}_t - \bar{X}\bar{H}(\hat{X}_t + \hat{H}_t) - \bar{K}[\delta \bar{Z}\hat{Z}_t + \hat{K}_{t+1}(\bar{Z} + \upsilon\delta) + \\ &\quad \hat{K}_t\{\bar{Z}(1 - \delta) + \left(\frac{\upsilon\delta}{2} \right)(2 - \delta)\}]. \end{aligned} \tag{B.2.42}$$

Now, recall from (B.2.12), (B.2.15) (or (B.2.40)) that

$$\hat{\Psi}_t = \left(\frac{\phi}{\sigma} \right) \left[\hat{W}_t + (\sigma - 1)\left(\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t + \psi_H \hat{H}_t \right) \right], \tag{B.2.43}$$

$$\hat{H}_t = (\nu_H)^{-1} \left[\hat{W}_t + (\sigma - 1)\left(\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t \right) - \sigma \hat{X}_t \right]. \tag{B.2.44}$$

But recognizing that $\hat{D}_t = d_t - \bar{d}$, substituting (B.2.43)-(B.2.44) and (B.2.29) (for $\hat{K}_{t+1} = k_{t+1} - k$) in (B.2.42), we can write the log of dividends as

$$d_t = N_{d,0} + N_{d,w} w_t + N_{d,\pi} \pi_t + N_{d,a} a_t + N_{d,x} x_t + N_{d,z} z_t + N_{d,k} k_t, \tag{B.2.45}$$

where

$$N_{d,w} = (\bar{D})^{-1}\left[\frac{\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}}{\nu_H}\right)-\bar{K}(\bar{Z}+\upsilon\delta)\xi_w\right] \tag{B.2.46}$$

$$N_{d,\pi} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)}{\nu_H}\right)-\bar{K}(\bar{Z}+\upsilon\delta)\xi_\pi\right] \tag{B.2.47}$$

$$N_{d,a} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)}{\nu_H}\right)-\bar{K}(\bar{Z}+\upsilon\delta)\xi_a\right] \tag{B.2.48}$$

$$N_{d,x} = -(\bar{D})^{-1}\left[\bar{\Psi}\phi\left(\frac{\psi_H}{\nu_H}\right)+\bar{X}\bar{H}+\bar{K}(\bar{Z}+\upsilon\delta)\xi_x\right] \tag{B.2.49}$$

$$N_{d,z} = -(\bar{D})^{-1}\bar{K}\left[\bar{Z}\delta+(\bar{Z}+\upsilon\delta)\xi_z\right] \tag{B.2.50}$$

$$N_{d,k} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\psi_K\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)\psi_K}{\nu_H}\right)-\bar{K}(\bar{Z}+\upsilon\delta)\xi_k+\bar{K}\left\{\bar{Z}(1-\delta)+\left(\frac{\upsilon\delta}{2}\right)(2-\delta)\right\}\right], \tag{B.2.51}$$

and $N_{d,0}$ is a term of steady state constants that will not affect the covariance function of the ERP that we consider below.

With log equilibrium dividends in hand, we return to equilibrium equity return condition (7) to deduce the state representation of the log of the stock price $s_t = \log(S_t)$. We first rewrite the equilibrium condition as

$$\mathbb{E}_t\left[M_{t,t+1}\left(\frac{D_{t+1}}{S_t}+\frac{S_{t+1}}{S_t}\right)\right]=1. \tag{B.2.52}$$

Log-linearizing (B.2.52) we get (using $\bar{M}=\alpha$)

$$\alpha\mathbb{E}_t\left[\left(\frac{\bar{D}}{\bar{S}}\right)\left(1+\hat{M}_{t,t+1}+\hat{D}_{t+1}-\hat{S}_t\right)+\left(1+\hat{M}_{t,t+1}+\hat{S}_{t+1}-\hat{S}_t\right)\right]=1. \tag{B.2.53}$$

However, in the steady state $\bar{S}=\frac{\alpha\bar{D}}{1-\alpha}$, and hence

$$\alpha\left(\frac{\bar{D}}{\bar{S}}+1\right)=\alpha\left(\frac{1-\alpha}{\alpha}+1\right)=1. \tag{B.2.54}$$

Therefore, (B.2.53) becomes (recognizing $\left(\frac{\bar{D}}{\bar{S}}+1\right)=1/\alpha$)

$$\mathbb{E}_t\left[\hat{M}_{t,t+1}+(1-\alpha)\hat{D}_{t+1}+\alpha\hat{S}_{t+1}-\hat{S}_t\right]=0. \tag{B.2.55}$$

We conjecture that

$$s_t = N_{s,0} + N_{s,w}w_t + N_{s,\pi}\pi_t + N_{s,a}a_t + N_{s,x}x_t + N_{s,z}z_t + N_{s,k}k_t, \tag{B.2.56}$$

so that

$$\begin{aligned}\mathbb{E}_t\left[\alpha\hat{S}_{t+1} - \hat{S}_t\right] &= (\alpha-1)(N_{s,0}-\bar{s}) + N_{s,w}(\alpha\rho_w - 1)w_t + N_{s,\pi}(\alpha\rho_\pi - 1)\pi_t + \\ &\quad N_{s,a}(\alpha\rho_a - 1)a_t + N_{s,x}(\alpha\rho_x - 1)x_t + \\ &\quad N_{s,z}(\alpha\rho_z - 1)z_t + N_{s,k}(\alpha k_{t+1} - k_t). \end{aligned} \tag{B.2.57}$$

Meanwhile, from the foregoing

$$\begin{aligned}\mathbb{E}_t\left[(1-\alpha)\hat{D}_{t+1}\right] &= (1-\alpha)\left[N_{d,0} + N_{d,w}\rho_w w_t + N_{d,\pi}\rho_\pi\pi_t + N_{d,a}\rho_a a_t + \right. \\ &\quad \left. N_{d,x}\rho_x x_t + N_{d,z}\rho_z z_t + N_{d,k}k_{t+1} - \bar{d}\right], \qquad \text{(B.2.58)} \\ \mathbb{E}_t\left[\hat{M}_{t,t+1}\right] &= (\theta-1)\log\alpha + B_w w_t + (B_\pi + (1-\rho_\pi))\pi_t. \qquad \text{(B.2.59)}\end{aligned}$$

But using (B.2.29), we have

$$\begin{aligned}\alpha k_{t+1} - k_t &= \alpha\left[\xi_0 + \xi_w w_t + \xi_\pi\pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t\right] + (\alpha\xi_k - 1)k_t, \qquad \text{(B.2.60)} \\ (1-\alpha)k_{t+1} &= (1-\alpha)\left[\xi_0 + \xi_w w_t + \xi_\pi\pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t + \xi_k k_t\right]. \qquad \text{(B.2.61)}\end{aligned}$$

Thus substituting (B.2.60)-(B.2.61) in (B.2.57)-(B.2.58) we get (up to constants),

$$\mathbb{E}_t\left[\hat{M}_{t,t+1} + (1-\alpha)\hat{D}_{t+1} + \alpha\hat{S}_{t+1} - \hat{S}_t\right] =$$

$$\begin{aligned}0 &= w_t\left(B_w + N_{s,w}(\alpha\rho_w - 1) + (1-\alpha)N_{d,w}\rho_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \pi_t\left(B_\pi + (1-\rho_\pi) + \right. \\ &\left. N_{s,\pi}(\alpha\rho_\pi - 1) + (1-\alpha)N_{d,\pi}\rho_\pi + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + a_t\left(N_{s,a}(\alpha\rho_a - 1) + (1-\alpha)N_{d,a}\rho_a + \right. \\ &\left. \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + x_t\left(N_{s,x}(\alpha\rho_x - 1) + (1-\alpha)N_{d,x}\rho_x + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \\ &z_t\left(N_{s,z}(\alpha\rho_z - 1) + (1-\alpha)N_{d,z}\rho_z + \xi_z(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \\ &k_t\left(N_{s,k}(\alpha\xi_k - 1) + (1-\alpha)N_{d,k}\xi_k\right).\end{aligned} \tag{B.2.62}$$

Hence, to ensure that items multiplying k_t in (B.2.63) are zero, we must have

$$N_{s,k} = \frac{(1-\alpha)N_{d,k}\xi_k}{1-\alpha\xi_k}, \tag{B.2.63}$$

where $N_{d,k}$ is computed in (B.2.50). With $N_{s,k}$ in hand, we can choose the remaining coefficients in (B.2.56) to ensure that (B.2.55) holds for all values of the state variables. That is,

$$
\begin{aligned}
N_{s,w} &= \frac{B_w + (1-\alpha)N_{d,w}\rho_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_w}, \\
N_{s,\pi} &= \frac{B_\pi + (1-\rho_\pi) + (1-\alpha)N_{d,\pi}\rho_\pi + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_\pi}, \\
N_{s,a} &= \frac{(1-\alpha)N_{d,a}\rho_a + \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_a}, \\
N_{s,x} &= \frac{(1-\alpha)N_{d,x}\rho_x + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_x}, \\
N_{s,z} &= \frac{(1-\alpha)N_{d,z}\rho_z + \xi_z(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho z},
\end{aligned}
\tag{B.2.64}
$$

Finally, the constant term in (B.2.56) can be computed to be

$$
N_{s,0} = \frac{(\alpha-1)(N_{d,0} - \bar{d} + \bar{s} + N_{d,k}\xi_0) - \alpha N_{s,k}\xi_0 - (\theta-1)\log\alpha}{(\alpha-1)}. \tag{B.2.65}
$$

The coefficients in (B.2.64)-(B.2.65) verify that s_t is indeed an affine function of the state variables.

Next, the Euler condition for equity returns imply (in the standard fashion)

$$
\mathbb{E}_t\left[R_{t+1} - R^f_{t+1}\right] = -Cov_t\left(M_{t,t+1}, \frac{D_{t+1} + S_{n,t+1}}{S_t}\right) R_{f,t+1}, \tag{B.2.66}
$$

or dividing both sides of (B.2.66) by $R_{f,t+1}$

$$
\mathbb{E}_t\left[\frac{R_{t+1}}{R^f_{t+1}} - 1\right] = -Cov_t\left(M_{t,t+1}, \frac{D_{t+1} + S_{t+1}}{S_t}\right). \tag{B.2.67}
$$

Log-linearizing the left hand side of (B.2.67) gives (for $r_{t+1} = \log(R_{t+1})$, $r^f_{t+1} = r^f_{t+1}$)

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{R_{t+1}}{R^f_{t+1}} - 1\right] &= \mathbb{E}_t\left[\left(\frac{R}{R^f}\right)\left(1 + \hat{R}_{t+1} - \hat{R}^f_{t+1}\right) - 1\right] \\
&= \mathbb{E}_t\left[r_{t+1} - r^f_{t+1}\right],
\end{aligned}
$$

since in the deterministic steady state $\frac{R}{R^f} = 1$ and hence $\hat{R}_{t+1} - \hat{R}^f_{t+1} = r_{t+1} - r^f_{t+1}$. Then log-linearizing both sides of (B.2.67) yields

$$\begin{aligned}\mathbb{E}_t\left[r_{t+1}-r^f_{t+1}\right] &= -Cov_t\left(\alpha(1+\hat{M}_{t,t+1}), \frac{\bar{D}}{\bar{S}}(1+\hat{D}_{t+1})+(1+\hat{S}_{t+1}-\hat{S}_t\right)\\ &= -\alpha Cov_t\left(\hat{M}_{t,t+1}, \frac{\bar{D}}{\bar{S}}\hat{D}_{t+1}+\hat{S}_{t+1}\right)\\ &= -\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1}, \left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right), \qquad \text{(B.2.68)}\end{aligned}$$

where the final equality follows from recognizing that $\hat{M}_{t,t+1}=m_{t+1}-\log\alpha, m_{t+1}=\lambda_{t+1}+\pi_t-\pi_{t+1}, \hat{D}_{t+1}+\hat{S}_{t+1}=d_{t+1}+s_{t+1}-\log(\bar{D})-\log(\bar{S})$, and $\bar{S}=\frac{\alpha\bar{D}}{1-\alpha}$. But

$$-\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1}, \left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right)=\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1}, -\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right\}\right)$$

$$=\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1}-\mathbb{E}_t[\lambda_{t+1}-\pi_{t+1}], -\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}-\mathbb{E}_t[\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}]\right\}\right). \qquad \text{(B.2.69)}$$

From (B.2.4) and (B.2.46)-(B.2.65) we get (recognizing that k_{t+1} is deterministic conditional on Γ_t)

$$\lambda_{t+1}-\pi_{t+1}-\mathbb{E}_t[\lambda_{t+1}-\pi_{t+1}]=b_w\varepsilon^w_{t+1}+(b_\pi-1)\varepsilon^\pi_{t+1}, \qquad \text{(B.2.70)}$$

$$-\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}-\mathbb{E}_t\left[\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right]\right\}=-\left(\left(\frac{1-\alpha}{\alpha}\right)\mathbf{N}_d+\mathbf{N}_s\right)'\boldsymbol{\varepsilon}_{t+1}, \qquad \text{(B.2.71)}$$

where b_w and b_π are defined in (B.2.5). Meanwhile, $\mathbf{N}_d=(N_{d,w}, N_{d,\pi}, N_{d,a}, N_{d,x}, N_{d,z})$ are defined in (B.2.46)-(B.2.51) and $\mathbf{N}_s=(N_{s,w}, N_{s,\pi}, N_{s,a}, N_{s,x}, N_{s,z})$ are defined in (B.2.64). An additional piece of notation allows one to more concisely express (B.2.69). Put

$$\tilde{N}_w=-\left[\left(\frac{1-\alpha}{\alpha}\right)N_{d,w}+N_{s,w}\right],$$

and similarly for $\tilde{N}_\pi, \tilde{N}_a, \tilde{N}_x$ to get from (B.2.46)-(B.2.51) and (B.2.64) (upon switching the sign of the denominator)

$$\tilde{N}_w = \frac{(1-\alpha)N_{d,w} + \alpha(B_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k}))}{\alpha(\alpha\rho_w - 1)}, \tag{B.2.72}$$

$$\tilde{N}_\pi = \frac{(1-\alpha)N_{d,\pi} + \alpha(B_\pi + (1-\rho_\pi) + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k}))}{\alpha(\alpha\rho_\pi - 1)}, \tag{B.2.73}$$

$$\tilde{N}_a = \frac{(1-\alpha)N_{d,a} + \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_a - 1)}, \tag{B.2.74}$$

$$\tilde{N}_x = \frac{(1-\alpha)N_{d,x} + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_x - 1)}, \tag{B.2.75}$$

$$\tilde{N}_z = \frac{(1-\alpha)N_{d,z} + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_z - 1)}. \tag{B.2.76}$$

Then,

$$\alpha Cov_t\left(\lambda_{t+1} - \pi_{t+1}, -\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1} + s_{t+1})\right\}\right) =$$

$$b_w\tilde{N}Var_t(\varepsilon^w_{t+1}) + (b_\pi - 1)\tilde{N}_\pi Var_t(\varepsilon^\pi_{t+1}) + ((b_\pi - 1)\tilde{N}_w + b_w\tilde{N}_\pi)Cov_t(\varepsilon^w_{t+1}, \varepsilon^\pi_{t+1}) + b_w\tilde{N}_a Cov_t(\varepsilon^w_{t+1}, \varepsilon^a_{t+1}) + (b_\pi - 1)\tilde{N}_a Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^a_{t+1}) + b_w\tilde{N}_x Cov_t(\varepsilon^w_{t+1}, \varepsilon^x_{t+1}) + (b_\pi - 1) \times \tilde{N}_x Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^x_{t+1}) + b_w\tilde{N}_z Cov_t(\varepsilon^w_{t+1}, \varepsilon^z_{t+1}) + (b_\pi - 1)\tilde{N}_z Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^z_{t+1}) \tag{B.2.77}$$

Hence, we can write

$$\begin{aligned}\mathbb{E}_t[r_{t+1} - r^f_{t+1}] &= \beta_w\Phi^2_w + \beta_\pi\Phi^2_\pi + \beta_{w\pi}\Phi_{w\pi} + \beta_{aw}\Phi_{aw} + \beta_{a\pi}\Phi_{a\pi} + \\ &\quad \beta_{xw}\Phi_{xw} + \beta_{x\pi}\Phi_{x\pi} + \beta_{zw}\Phi_{zw} + \beta_{z\pi}\Phi_{z\pi},\end{aligned} \tag{B.2.78}$$

where

$$\begin{aligned}\beta_w &= \alpha b_w\tilde{N}_w, \beta_\pi = \alpha(b_\pi - 1)\tilde{N}_\pi, \beta_{w\pi} = \alpha((b_\pi - 1)\tilde{N}_w + b_w\tilde{N}_\pi), \\ \beta_{aw} &= \alpha b_w\tilde{N}_a, \beta_{a\pi} = \alpha(b_\pi - 1)\tilde{N}_a, \beta_{xw} = \alpha b_w\tilde{N}_x, \\ \beta_{x\pi} &= \alpha(b_\pi - 1)\tilde{N}_x, \beta_{zw} = \alpha b_w\tilde{N}_z, \beta_{z\pi} = \alpha(b_\pi - 1)\tilde{N}_z.\end{aligned} \tag{B.2.79}$$

B.2.4 Effects of σ on sensitivity of optimal investment and production policies to productivity shocks

We start with the log-linearized equilibrium material input demand ($\hat{H}_t$) in (B.2.15) (also re-stated in (B.2.44)). Recalling that $\nu_H \equiv \psi_H + \sigma(1-\psi_H)$, the effect of σ on the sensitivity of $\hat{H}_t$ to the exogenous productivity shock $\hat{A}_t$ is

$$\begin{aligned}\frac{\partial}{\partial \sigma}\left(\frac{\partial \hat{H}_t}{\partial \hat{A}_t}\right) &= \frac{\partial}{\partial \sigma}\left(\frac{\sigma - 1}{\nu_H}\right) \\ &= \frac{1}{\nu_H^2} > 0. \end{aligned} \tag{B.2.80}$$

Hence, sensitivity of equilibrium production to productivity shocks is increasing with the ES.

Next, we examine the effect of σ on the sensitivity of equilibrium capital investment to the productivity shock. The log-linearized equilibrium capital investment policy (up to constants) is given in (B.2.28). Hence, we examine the effect of variations in σ on $\frac{\partial k_{t+1}}{\partial a_t} = \xi_k$. Now, recalling that $u_2^K = \upsilon > 0$, $u_0^K = \left(\frac{\upsilon}{\alpha}\right) > 0$, it follows from (B.2.30) that u_1^K and ξ_k must be of opposite sign. Meanwhile, we have from (B.2.26) that $u_1^K < 0$ if

$$\frac{\phi}{\sigma}(\sigma - 1)\psi_K(1 + u^H) < \sigma. \tag{B.2.81}$$

Now, if the number of sectors is large, then the typical individual sector weight ϕ is small. Furthermore, $\sigma > 1, \psi_K < 1$, and $u^H = \frac{\psi_H}{\psi_H + \sigma(1-\psi_H)} < \psi_H < 1$. Hence, for reasonable calibrations, $u_1^K < 0$. But then (B.2.31) implies that

$$\xi_k = -\frac{u_1^K}{2u_2^K} - \frac{\sqrt{(u_1^K)^2 - 4u_2^K u_0^K}}{2u_2^K}, \tag{B.2.82}$$

is the smaller, non-explosive root.

We then have,

$$\frac{\partial \xi_k}{\partial \sigma} \propto -\frac{\partial u_1^K}{\partial \sigma} - \frac{\frac{\partial u_1^K}{\partial \sigma}}{\sqrt{(u_1^K)^2 - 4u_2^K u_0^K}}. \tag{B.2.83}$$

We compute then from (B.2.31),

$$\frac{\partial u_1^K}{\partial \sigma} \propto \frac{\phi \psi_K (1 + u^H)}{\sigma^2} - 1 < 0, \tag{B.2.84}$$

for reasons given above. In fact, (B.2.84) will apply for all model parameterzations with $\sigma \geq \sqrt{2}$. It then follows from B.2.84) that $\frac{\partial \xi_k}{\partial \sigma} > 0$.

Finally, we compute from (B.2.35),

$$\begin{aligned}\frac{\partial \xi_a}{\partial \sigma} \propto -\Bigg[&(u_2^K(\rho_a + \xi_k) + u_1^K)\frac{\phi \psi_K \bar{\Psi} \rho_a}{\bar{K}}\left(\nu_H(\psi_H + \nu_H) - \sigma(\sigma - 1)(1 - \psi_H)\right) - \\ &\left(\frac{\phi \psi_K \bar{\Psi}(\sigma - 1)\rho_a(1 + u^H)}{\sigma \bar{K}}\right)\left(\frac{1}{2}\frac{\partial u_1^K}{\partial \sigma}\frac{(\zeta - 1)}{\zeta}\right)\Bigg], \end{aligned} \tag{B.2.85}$$

where $\zeta \equiv \sqrt{(u_1^K)^2 - 4u_2^K u_0^K}$. We note that $(u_2^K(\rho_a + \xi_k) + u_1^K) < 0$ from (B.2.35) since $\xi_a > 0$, i.e., equilibrium investment is procyclical. We can then show that $\frac{\partial \xi_a}{\partial \sigma} > 0$ for an open set of parameters with $\sigma > 1$ and $\psi_H < 1$.

C.1 Steady state parameterization

To match the endogenous $\bar{H}, \bar{K}$ and $\bar{Y}$ to the sample means of the industry data (in per capita terms), we proceed as follows. Let MC, Y_{data} denote the sample mean of the materials cost. Then, by definition, $MC = \bar{X}\bar{H}$. Hence, the materials input optimality condition (B.1.1) can be written (recognizing that $\psi_H \bar{Y}/\bar{H}$)

$$\frac{\bar{p}\bar{Y}}{\bar{H}} = \frac{MC}{\bar{H}}, \tag{C.1.1}$$

which implies that

$$\psi_H(\bar{p}\bar{Y}) = MC. \tag{C.1.2}$$

Meanwhile, the steady state Euler condition (B.1.2) implies

$$\psi_K\left(\frac{\bar{p}\bar{Y}}{\bar{K}}\right) = \left(\frac{\bar{Z} + \upsilon\delta}{\alpha}\right)(1 - \alpha(1-\delta)) - 0.5\upsilon(\delta)^2. \tag{C.1.3}$$

Put $e_K \equiv \left(\frac{\bar{Z}+\upsilon\delta}{\alpha}\right)(1 - \alpha(1-\delta)) - 0.5\upsilon(\delta)^2$. Now dividing (C.1.2) by (C.1.3) gives

$$\bar{K} = \left(\frac{\psi_K}{\psi_H}\right)\left[\frac{MC}{e_K}\right]. \tag{C.1.4}$$

Then we set

$$\bar{H} = \left(\frac{Y_{data}}{(\bar{K})^{\psi_K}}\right)^{1/\psi_H}, \tag{C.1.5}$$

where $\bar{K}$ is given in (C.1.4). $\bar{X}$ is then computed as the ratio $MC/\bar{H}$. And using $\bar{H}$ and $\bar{K}$ above, $\bar{A}$ is set such that

$$\bar{A} = \frac{Y_{data}}{(\bar{K})^{\psi_K}(H)^{\psi_H}}. \tag{C.1.6}$$

Finally, $\bar{Z}$, ψ_K, ψ_H are calibrated to match $\bar{H}, \bar{K}$ and $\bar{Y}$ with the sample mean values of industry data (in per capita terms).

Table A.1. Steady state values for numerical calculations

Steady State Values			
Aggregate	$\bar{W}$ (dollars)	$\bar{P}$	$\frac{\bar{C}}{\bar{W}}$
	19905	1.286	0.78 (0.78)
Sector	$\bar{Y}$ (million dollars)	$\bar{X}\bar{H}$(million dollars)	$\bar{Z}\bar{K}$ (million dollars)
Consumer Durables	18.23 (18.23)	9.56 (9.56)	10.38 (10.34)
Consumer Non-Durables	23.69 (23.69)	13.96 (13.96)	12.48 (12.71)

Notes to Table: This table displays the steady state aggregate and sectoral values used in the numerical calculations of sectoral equity risk premium (ERP) in Table 1. The steady state value for per capita aggregate personal income ($\bar{W}$) is the sample mean, and $\bar{P}$ is the sample mean of the per capita consumption-to-income ratio C/W. The sectoral production variables are the steady state values computed from the model, while the entries in parentheses are the corresponding sample means.